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Executive Summary

The deliverable D5.5 describes the experimental demonstration of the rate and transmission distance achievable by the coherent nonlinear Fourier-transform (NFT) based communication system done by the members of the consortium in the laboratories of Nokia Bell Labs. In more detail, a high-capacity dual-polarized nonlinear frequency-division transmission system, achieving a net data rate of 220 Gb/s with spectral efficiency of 4 bits/s/Hz has been experimentally demonstrated for the first time. The result was enabled by employing the concept of b-modulation and developing custom optimized digital signal processing algorithms.

To bring this research into a broader context, the deliverable reviews the general mathematical concept of NFT, its possible applications to the coherent optical telecommunications, and describes the transmission rates and distances reported for the numerical and experimental demonstrations of NFT-based communication systems done by the other groups working in the field. More specifically, D5.5 presents an overview of the possible designs of the NFT based optical data transmission. NFT can be implemented both as an extra processing block to the existing processing elements and as a new type of signal modulation. In particular, NFT can be used for the NFT-based digital backward propagation at the receiver or as a pre-compensator of both dispersive and nonlinear channel impairments at the transmitter. Nonlinear frequency division multiplexing (NFDM) assumes modulation of the nonlinear spectrum of the signal: continuous, discrete or both. The presented experimental results demonstrate the state-of-the-art achievements with using NFDM. Following recent explosion of interest in the machine learning based techniques, we also designed transmission system combining NFDM and machine learning methods. DTU has proposed and designed a new approach based on NFDM with pre-and post-processing of the optical signal.

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LIST OF ACRONYMS

AiPT	Aston Institute Of Photonic Technologies
EC	European Commission
EID	European Industrial Doctorates
ESR	Early Stage Researcher
FONTE	Fibre Optic Nonlinear Technologies
NFT	Nonlinear Fourier Transform
NFD	Nonlinear Fourier Domain
NS	Nonlinear Spectrum
DBP	Digital Back-Propagation
NIS	Nonlinear Inverse Spectrum
INFT	Inverse Nonlinear Spectrum
ML	Machine Learning
TX	Transmitter
RX	Receiver
NFDM	Modulation in nonlinear Fourier domain
QAM	Quadrature amplitude modulation
OFDM	Orthogonal Frequency Division Multiplexing
QPSK	Quadrature Phase-Shift Keying
DT	Darboux transform
CM-NFDM	Continuously-modulated NFDM

1 INTRODUCTION

Nonlinear Fourier transforms (NFT) can be implemented to develop the communication systems resistant to the nonlinear distortion. NFT term refers to the conversion of the time-domain optical signal to the specific domain, named as a nonlinear Fourier domain (NFD) or nonlinear spectral domain [1, 2]. The conversion result is typically named as the nonlinear spectrum (NS) of the signal. The main advantage of NS domain is that the effect of both the second-order chromatic dispersion (CD) and Kerr nonlinearity on the transmitted optical signal, usually complex to grasp in the time-domain, in NFD is represented as the pointwise signal-independent phase rotation of NS.

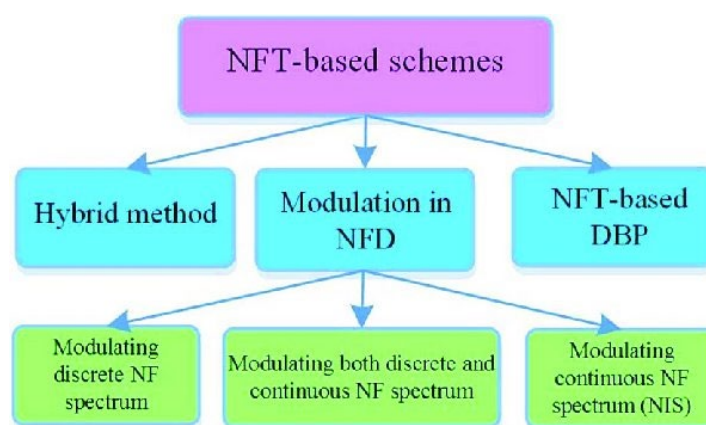


Figure 1: Diagram of the currently proposed and studied NFT-based methods. The figure is kindly provided by the authors of [3].

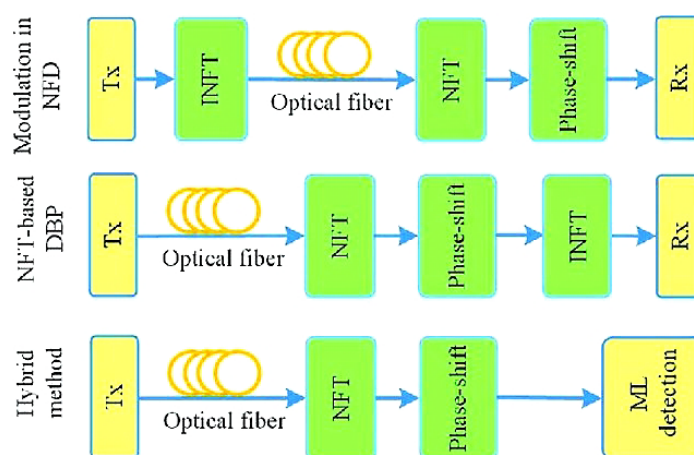


Figure 2: Basic designs of NFT-based transmission systems, including transmission in the NFT domain, DBP with the use of NFT operations, and the hybrid method. The figure is kindly provided by the authors of [3].

Since the evolution of individual NS modes is linear and decoupled, one can use these modes for coding, transmission, detection, and processing of information. By now, the three principal designs for NFT-based transmission systems were suggested [3]. These designs are schematically presented in the Figs. 1, 2. In the first approach, the transmitted information is encoded onto NS which is then converted by INFT to the time-domain optical signal propagated over the link. At the receiver (RX) the received optical signal is converted back from time-domain to NS, from which the information is demodulated later. This design is referred to as

modulation in the nonlinear Fourier domain (NFDm). In this paradigm, one can modulate the discrete [4] and continuous [5] parts of NS separately or together [6].

In the second approach, the NFT is employed to compensate for the nonlinear distortion in digital domain at the transmitter or receiver side. The advantage of this concept is that here NFTs is applied to calculate the inverse signal propagation in a way similar to the conventional digital back-propagation (DBP) approach [7], while the signal modulation is done conventionally (e.g, by QAM or OFDM) in the time domain [8,9]. Nonetheless, the proper implementation of the NFT-based DBP is hindered by a yet unsolved challenge – the difficulty of precise calculation of direct NFT caused by the a priori unknown number of discrete eigenvalues in the NS corresponding to a randomly modulated information-bearing signal. Therefore, by now, the NFT-based DBP approach is proposed only for the low-energy soliton-free case and for normal dispersion fibres [8, 9], where no discrete NS is formed. In [8] NFT-based DBP was successfully applied to the numerical model of 100-Gb/s QPSK and 300 Gb/s 64QAM transmission over 4000 km long link with the fibre with the normal chromatic dispersion. In [9] the authors limited themselves to a proof-of-concept demonstration.

In the third approach, denoted in Fig. 2 as the “hybrid method”, the information about the NS of the optical signal is used during decoding the signal at the receiver [10]. In [10] the demodulation of 32 Gb/s BPSK over 240 km link utilizing only information from the NS of the received signal was experimentally demonstrated.

The further discussion will be limited to the NFDm design, which is the currently the mainstream approach in application NFT to fibre-optic communications. Because the nonlinear spectrum consists of discrete and continuous part which can be separately modulated and decoded

2 NFDm UTILISING THE DISCRETE SPECTRUM MODULATION

Starting from the initial paper, brought by Hasegawa and Nyu [11], the discrete part of nonlinear spectrum (NS), corresponding in time-domain to the stable solitary pulses, was suggested as the main information bearer in NFDm systems. In this method the transmitted information can be coded by the presence of particular discrete eigenvalues in the NS of the transmitted signal. Another way is to modulate the complex-valued residues of the poles NS acquire in the eigenvalue points. Several milestone technologies significantly improved the transmission quality of the systems utilising discrete NS.

First, INFT calculation for the discrete spectrum with several eigenvalues was significantly simplified by the application of Darboux transformation (DT). This transformation provides an analytical description of how the time-domain form of a discrete NS changes if a new eigenvalue is added to it. By thus, DT allowed to replace the complex INFT calculation of multi-eigenvalue NS with the simpler task of first converting single-eigenvalue NS to the time-domain and then modifying the left spectrum by a Darboux transform. The first application of DT to constructing the temporal profile corresponding to the multi-eigenvalue nonlinear spectrum was proposed in [12]. Next, the further simplification of DT-based procedure of the multi-eigenvalue solution construction was suggested in [13]. Finally, the fast numerical DT computation algorithm with the $O(N \log^2 N)$ cost was proposed in [14].

Second, an important advancement in utilizing the discrete NS for NFDm-based communications is the forward-backward (FB) algorithm [13, 15] of retrieving the discrete NS corresponding to the given time-domain signal. The FB algorithm offers more precise calculation of the discrete spectrum eigenvalues restore accurate enough the phase and amplitude values of discrete spectral components corresponding to them.

Third, the authors of [16] shown that all the information regarding amplitude and phase of the discrete NS component $r(\xi_d) = b(\xi_d)/a'(\xi_d)$, is contained the spectral b-coefficient $b(\xi_d)$ and, hence, it can be modulated directly instead of $r(\xi_d)$. This allows us to exclude from the demodulation procedure the noise injected into $a'(\xi_d)$ spectrum component caused by the divergence of real fibre-optic channel from the ideal NLSE model assumed in NFDm approach. This noise is referred to as the divergence noise.

Fourth, in [16] it was also shown that the divergence noise present in the detected $b(\xi_d)$ value is correlated with the one distorting the eigenvalue ξ_d and the other discrete spectrum component corresponding to it $a'(\xi_d)$. Therefore, [16] proposed using the information about the noise detected in ξ_d and $a'(\xi_d)$ to decrease the noise contribution in $b(\xi_d)$.

Finally, the extension to the case of dual-polarized transmission of NFT procedure originally developed for the single-polarization NFT was proposed in [26]. This procedure allowed to utilize both polarizations for the signal transmission.

The aforementioned technologies let the several performance milestones to be achieved for NFDm systems operating strictly with the discrete spectrum. The transmission lengths and the information rates, achieved by the proposed pioneering discrete spectrum systems [19, 26–31] are compared in Fig. 3.



Figure 3: The performance obtained by the pioneering discrete NFDm communication systems. The figure is kindly provided by the author of [24].

3 NFDM UTILISING MODULATION OF BOTH DISCRETE AND CONTINUOUS NONLINEAR SPECTRUM

Originally, in view of [11] only the discrete part of nonlinear spectrum was supposed to be used in NFDM-based communication systems. Notwithstanding this, the authors of [5] proposed the NFDM system with the information being encoded onto the continuous part of the nonlinear Fourier spectrum. The main advantage of continuous NS is that usually it can bear much more information than the discrete one [25]. Unfortunately, continuous NS causes a problem non-present in a discrete NS: in time-domain continuously modulated NFDM (CM-NFDM) signal broaden during their propagation over the fibre-optic link because of chromatic dispersion. Therefore, the adjacent CM-NFDM-modulated pulses have to be separated in time domain by empty guard intervals to prevent the destructive inter-pulse interference during their propagation. The interval duration of the intervals must be no shorter than the channel memory, induced by chromatic dispersion. The presence of guard intervals, carrying no useful information limits the spectral efficiency of the continuous-spectrum-based NFDM communication systems. Hence, all the major improvements in the efficiency of CM-NFDM systems were reached either by shortening time-width of the pulses or of the guard intervals separating them.

First, in [26] it was shown by pre-compensating one-half of the transmission-induced phase shift of the NS at the transmitter and compensating another half at the receiver decrease width of the required guard interval in two times.

Second, the authors of [27, 28] showed that the width of CM-NFDM pulses in time-domain can be also decreased by applying the specific modulation to spectral coefficient $b(\xi)$ instead of conventional modulation of full nonlinear spectral coefficient $r(\xi) = b(\xi)/a(\xi)$. The authors coined the term b -modulation for this approach. Notably, it was reported that the b -modulation allows up to two-fold reduction of the temporal width of CM-NFDM compared to conventional modulation approach.

Finally, in [29] expanded originally single-polarized b -modulated CM-NFDM system for the dual-polarization transmission, by thus significantly improving the spectral efficiency.

The modern NFDM systems typically combine the modulation of both discrete and continuous nonlinear spectrum. The aforementioned Darboux transform [13], was applied by the authors of [30] to expanding a time-domain signal, conveying purely continuous nonlinear spectrum, with a multi-eigenvalue discrete spectrum. This proposal led to the development of combined continuous-and-discrete NFDM systems, modulating the information onto all possible degrees of freedom of the nonlinear spectrum.

Fig. 4 describes the performance achievements reached by various NFDM communication systems prototypes utilizing modulation of the continuous NS with or without the discrete NS.

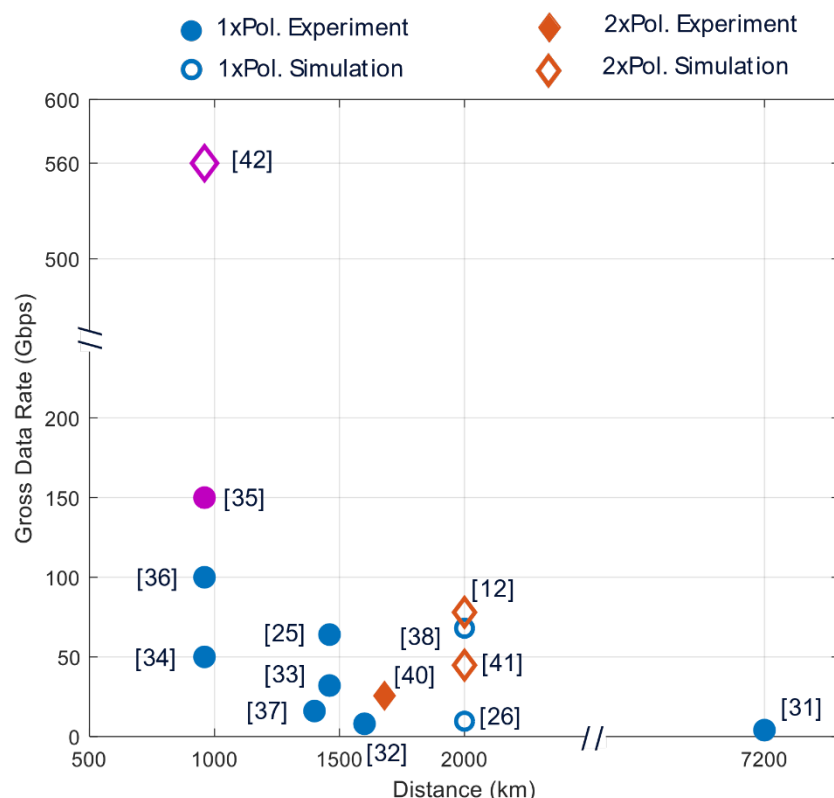


Figure 4: The performance obtained by the pioneering continuous and dual continuous-and-discrete NFD communication systems. The figure is kindly provided by the author of [20].

Notably, the b-modulated dual-polarisation NFD system, illustrated numerically in [42], was experimentally verified in [43].

For the transmission experiment, at transmitter, four digital-analog converters and a polarisation-division multiplexing I/Q modulator were used to generate the waveform. The output signal was subsequently amplified by an EDFA. Signals were then coupled in and out of the recirculating loop by a 50:50 coupler and travel four loops in the experiment. The transmission of dual-polarized (DP) CM-NFD signals of roughly 55 GHz (NC/T0) linear bandwidth was considered. The guard interval between DP-NFD symbols and their width was estimated as ≈ 3.57 ns. Each loop had three spans of SMF-28 (Corning), each 81.3 km length, and three EDFAs, making the total distance of the experiment 975.6 km. As receiver, preceded by another EDFA, an oscilloscope (DSOZ334A) with four channels was used. The sampled data taken by the oscilloscope were saved and processed offline. Two external cavity lasers were used as the optical carrier at the transmitter and as the local oscillator at the receiver. The wavelength selective switch functioned as an optical band-pass filter to remove the out-of-band amplified spontaneous emission noise.

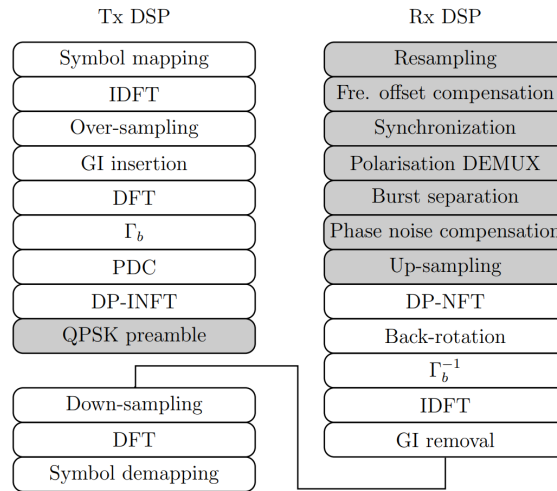


Figure 5: The transceiver digital signal processing chain utilized in [43]. The figure is kindly provided by the author of [43].

Several DSP blocks, marked in grey colour in Fig. 5, were added to deal with synchronisation, frequency-offset compensation, arbitrary polarisation rotation and tracking, and phase noise of the laser. In the transmission experiment the waveforms were launched at their designed launch powers, and the polarisation state was also randomly changed at the transmitter. The scheme of the used experimental setup is given in Fig. 6.

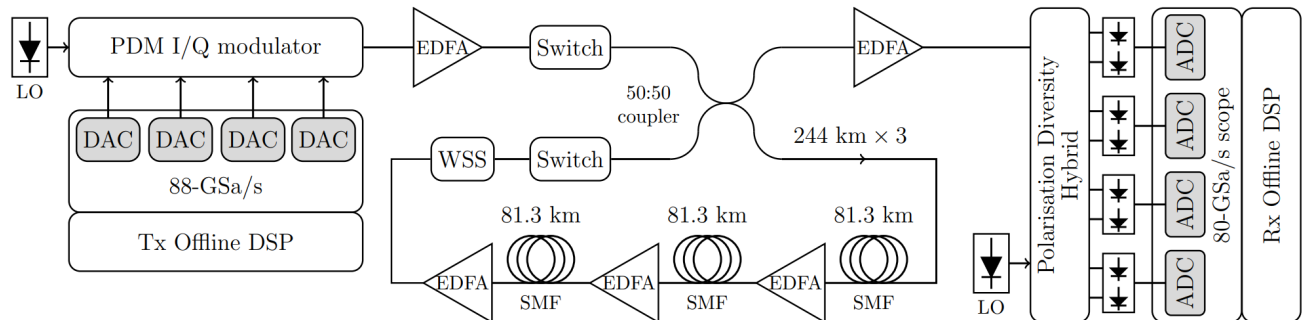


Figure 6: The recirculating fibre loop experiment setup utilized in [43]. The figure is kindly provided by the author of [43].

To compute the spectral efficiency, the authors of [43] estimated the mutual Information (of individual sub-carrier detection) by assuming the channel conditional distribution as a Gaussian distribution [44]. The net data rate was achieved by utilizing a soft-decision capacity approaching binary code such as spatially coupled low-density parity-check codes [45]. At the launch power of -1.3 dBm, the system reached its maximum net rate of 220Gbps, with the spectral efficiency of 4 bits/s/Hz.

5 NFDm WITH PRE- AND POST PROCESSING

The field of NFT is still developing and new ideas and concepts are generated continuously. We would like to mention work performed at DTU on NFDm with using both pre- and post-processing. This approach is based on combining NFDm with a recently proposed ML approach called an autoencoder [46]. An autoencoder consists of two key transformations: an encoder function that maps the input data to a code i.e. an encoded version of the data, and a decoder function that tries to reconstruct the original data from the code. Given the

similarities between the structure of an autoencoder and of a communication system, it was proposed to replace the transmitter with an encoder and the receiver with a decoder, which can then be jointly trained over the transmission channel. We applied this concept to jointly train an NFDm-based transmitter with a NN-receiver over a practical optical channel, i.e. including loss and noise. The high-level simulated setup is shown in Fig. 7. The rationale behind not fully replacing the transmitter with a complete NN was to make use of the theoretical understanding that soliton-like waveforms are theoretical solutions of the lossless optical fibre. They are therefore considered a relevant starting point for guiding the optimization over a practical lossy fibre.

Different level of optimization, i.e. of optimization parameters, were considered and benchmarked and the results are shown in Fig. 7(b), overall showing more than a factor 3 improvement in transmission reach.

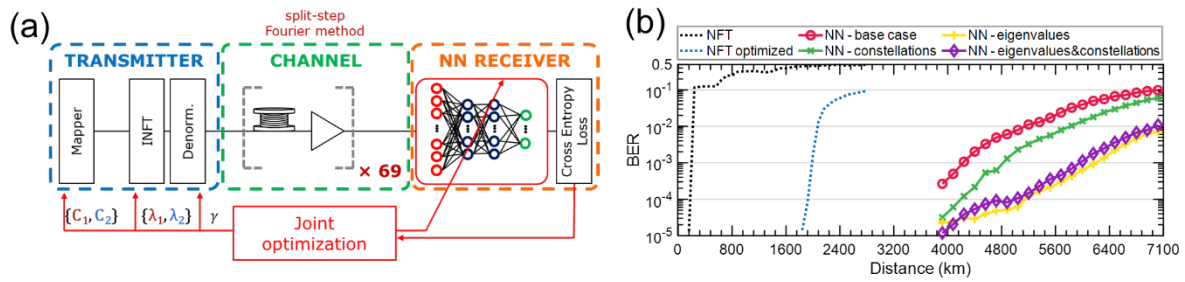


Figure 7: Design of advanced setup for joint pre- and post-processing optimization of NFDm transmission, and (b) transmission performance of different optimization scenarios benchmarked against standard NFDm transmission systems.

The experimental validation in a lab testbed, has been performed for the post-processing scheme. Note that modelling did not include several important effects that have been observed in the experiments. One of these effects is laser phase noise which adds memory to the received waveforms. In order to account for laser phase noise, an ad-hoc carrier phase recovery algorithm has been developed by modifying a standard digital phase-lock-loop approach with a phase estimator based itself on a NN. The reason for moving to a NN-based phase estimator was dictated by the need for recovering an averaged phase drift over the symbol solitonic waveform. Following on the properties of the NFDm, such an average phase drift could be directly compensated in the time domain. Once such a NN-based carrier recovery was trained offline with simulated data, it was applied to the experimentally measure waveforms, which were partly used to train the NN-based symbol detector, and partly for testing the overall system performance.

The experimental setup considered for the validation of the post-processing scheme is shown in Fig. 8(a) together with its performance in Fig 8(b). Fig 8(b), compares the NN receiver with a standard NFDm receiver.

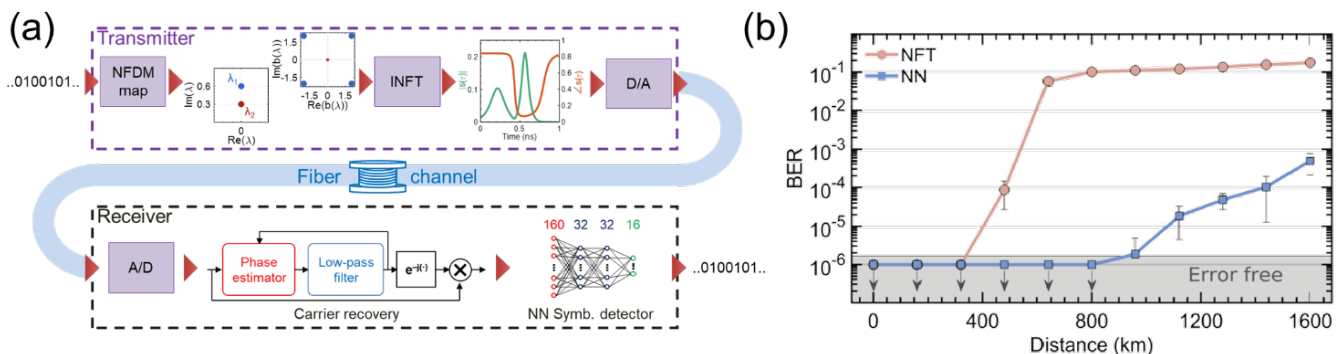


Figure 8: (a) high-level description of experimental testbed including signal processing blocks at transmitter and receiver and (b) transmission performance comparison between NN-receiver and NFT receiver [47].

These experiments clearly show the trend of a substantially longer transmission reach achievable by the NN receiver is clearly visible. We believe that future development of NFT and NFDMM will include pre- and post-processing based on the machine learning techniques.

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