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Executive Summary

Within this deliverable we review the advanced modulation techniques which can be employed for the NFT-based transmission systems. In particular, we address the case when both continuous **spectrum** (the quasi-linear part of nonlinear spectrum) and discrete eigenspectrum (the solitonic part) are modulated. This type of modulation can be achieved by the simultaneous application of Darboux transform and the numerical solution of Gelfand-Levitan-Marchenko equations. Then, we review the advances in the most recent nonlinear frequency-division multiplexing (NFDM) technique, which allows us to have the explicit control over the duration of the generated signal: the b-modulation. As it is shown, with the use of exponential scaling for b-function, we can achieve 400 Gbit/s data rate for the dual-polarisation case, which is the up-to-date number obtained by any NFDM system.

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LIST OF ACRONYMS

AWGN	Additive white Gaussian noise
AiPT	Aston Institute Of Photonic Technologies
B2B	Back-to-back
DBP	Digital back-propagation
DP	Dual-polarisation
EC	European Commission
EID	European Industrial Doctorates
ESR	Early Stage Researcher
FONTE	Fibre Optic Nonlinear Technologies
GLM	Gelfand-Levitan-Marchenko
I/Q	In-phase/quadrature
INFT	Inverse nonlinear Fourier transform
ML	Machine learning
NF	Nonlinear Fourier
NFD	Nonlinear Fourier domain
NFDM	Nonlinear frequency-division multiplexing
NFT	Nonlinear Fourier transform
NIS	Nonlinear inverse synthesis
NLS	Nonlinear Schrödinger equation
NS	Nonlinear spectrum
OA	Optical amplifier
OFDM	Optical frequency-division multiplexing
PNFT	Periodic nonlinear Fourier transform
Rx	Receiver
Tx	Transmitter

1 Introduction

In many different physical areas, and, particularly, in fiber optics, the signal's evolution can often be well approximated by the nonlinear Schrödinger equation (NLS) [1, 2]. In particular, the latter serves as a leading order model that describes the propagation of light envelope in fiber-optic communication channels under some simplifying conditions [1]. The normalised lossless and noiseless NLS for a slowly varying complex electromagnetic field envelope function $u(z, t)$, where z is the distance along the fibre and t is the retarded time (in the fibre optics context), is given as follows

$$iu_z + u_{tt} + 2|u|^2 u = 0\,, \tag{1}$$

i is the imaginary unity; for the explicit normalisations pertaining to single-mode fibres optical communications, see, e.g., [3]. The important property of NLS (1) is that it belongs to the class of the so-called integrable equations, meaning that the initial-value problem for this equation can be solved by means of inverse scattering technique [3, 4], given some constraints on the "initial conditions", $u(0, t)$ in our notations. The signal processing operations participating in this method are often referred to as the nonlinear Fourier transform (NFT), and the multiplexing technique dealing with the nonlinear Fourier (NF) domain data was coined nonlinear frequency division multiplexing [5]. In a nutshell, the NFT maps the solution of NLS (i.e. time-domain signal) onto the space of the complex-valued spectral parameter k, playing the role of a "nonlinear frequency", such that the NFT operation, decomposes our space-time profile into the nonlinear modes evolving inside the NF domain. The nonlinear spectrum (i.e. the "NFT image") that corresponds to the initial profile with a finite first norm, $u(0, t) \in L^1(\mathbb{R})$, contains, in the general case:

1. two scattering coefficients $a(\xi)$, $b(\xi)$ for $\xi \in \mathbb{R}$ describing the dispersive radiation components of our pulse, referred to as the *continuous spectrum*

2. *the discrete (solitonic) spectrum*, consisting of two complex parameters for each discrete (soliton) mode: the eigenvalue ξ_j and the respective spectral amplitude c_j.

The continuous spectrum is often represented through the relation of the scattering coefficients $q(\xi) = b(\xi)/a(\xi)$.

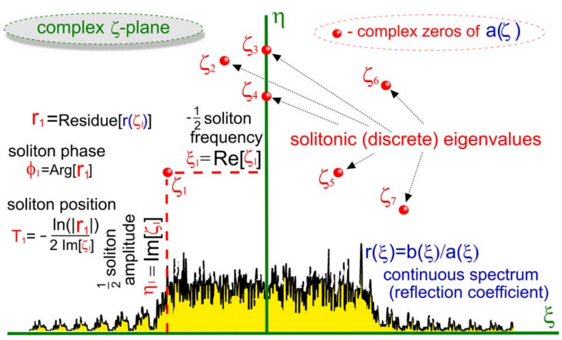

Figure 1: Exemplary NF spectrum (anomalous dispersion case), contain- ing solitons (discrete eigenvalues) and continuous nonlinear spectrum (depicted on the real axis ξ). Continuous spectrum is denoted on figure as $r(\xi)$, while refered to in the work as $q(\xi)$. The figure was originally published in [3].

Either discrete or continuous part of the NF spectrum can be absent in some specific situations [3, 5]. Since the NF modes evolve linearly inside the NF domain, the NFT-based optical signal processing and the usage of the parameters of nonlinear modes as data carriers have been considered as an efficacious method for the nonlinearity mitigation in optical fibre links [3].

Three main approaches were suggested for NFT-based transmission system. They are schematically presented in Figs. 3 and 4. First, the information can be encoded directly onto the NF signal spectrum, and then converted to the time-domain via inverse nonlinear Fourier transform (INFT) for the later transmission. One can understand it as a "modulation in the NF domain." Particularly, the discrete [7–9] and continuous [10, 11] NF spectrum can be modulated separately or together [6, 12]. Second, the NFT can be employed solely for the nonlinear mitigation on the receiver. In this approach, NFT is used to perform digital back propagation (DBP) [13] at the receiver in a numerically cheap single linear operation [14, 15]. The main benefit of this approach is that one can use conventional modulation and signal encoding schemes [16]. Nonetheless, because of the necessity to accurately calculate an unknown number of discrete eigenvalues in an arbitrary information-bearing signal NFT-based DBP was implemented by far only for the cases with the soliton-free NS at the receiver [3]. Third, one can encode the information at transmitter conventionally in time domain, but make decision at the receiver using NF spectrum data obtained via NF operations implemented there [17–21]. This method is referred to as "hybrid method".

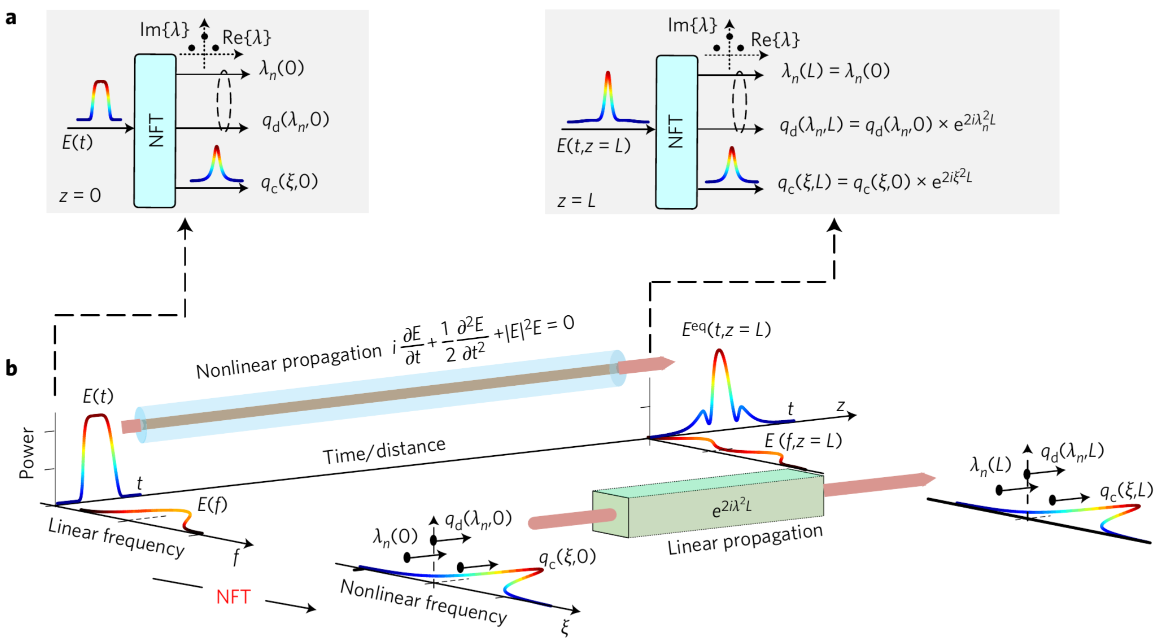

Figure 2: Illustration of the nonlinear spectrum and its evolution along a fibre link. **a**, Decomposition of the signal into nonlinear spectral data (nonlinear Fourier spectrum) through the NFT. All the interplay of dispersion and nonlinearity in the fibre link can be expressed by a simple linear change of phase in the nonlinear Fourier spectrum. **b**, Two-dimensional illustration of the time and frequency domain evolution of the third-order super-Gaussian pulse when chromatic dispersion is fully compensated and the nonlinear Fourier spectrum evolution of $E(t)$ (the discrete part denoted by black dots and the continuous part ($q_c(\xi)$) along the fibre link). The figure was originally published in [6].

2 Nonlinear freqency-division multiplexing systems

In this work, we concentrate on the first approach, referred to as the nonlinear freqency-division multiplexing (NFDM) transmission system.

In an NFDM system, the transmitted information-bearing signal is modulated in the nonlinear Fourier domain, in other words on the nonlinear spectrum. First, as shown in Fig. 6, we encode information directly on the nonlinear spectrum by modulating its discrete and/or continuous parts. After, the modulated NS is converted to the time-domain via the inverse NFT implemented through solving the Gelfand–Levitan–Marchenko (GLM) equation [5, 22]. Next, the obtained time-domain signal is carved out of the continuous waveform laser radiation by an in-phase/quadrature (I/Q) modulator. The resulting optical signal is sent into a fiber link. At the receiver, the collected signal is deformed by a complex nonlinear distortion arisen along the link from the interplay of linear memory introduced by chromatic dispersion and the Kerr nonlinearity. This distortion is compensated in the nonlinear Fourier domain by a single phase-shift [5, 10]. Therefore, these deterministic nonlinear distortions do not affect the recovery of the information encoded into the nonlinear spectrum.

Accidentally, the nonlinear Fourier spectrum can be calculated only for time-domain signals decaying to zero at the boundary, because of the vanishing boundary conditions of the inverse scattering technique [3, 4, 23, 24]. Therefore, in NFDM systems the signals are sent with guard empty time-bands between neighbouring information-bearing signals, in other words, bursts. Furthermore, these intervals have to be longer than the dispersion-induced channel memory in order to prevent bursts interfering with each other during propagation.

3 Advanced modulation techniqies for NFDM

The excessive length of bursts is a limiting factor in NFDM communications, since it imposes the upper limit on the capacity of the link. Too long bursts decrease the optimal transmission power of the NFDM package and by thus makes it more susceptible to the AWGN noise, injected by optical amplifiers and transceivers [25, 26]. Nonetheless, in classical NFT-based transmission [3, 11] modulation methods (i.e., methods to convert blocks of information into nonlinear Fourier spectrum signals) didn't not offer tight control over the length of the resulting time-domain pulse.

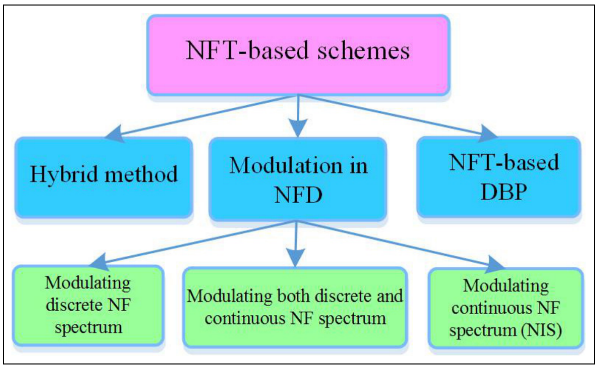

Figure 3: Diagram of the currently proposed and studied NFT-based methods. The figure was originally published in [3].

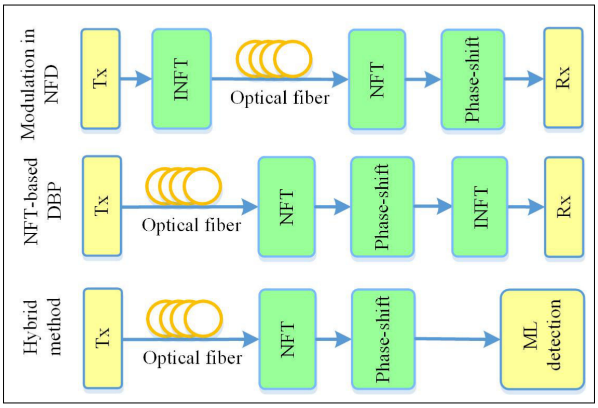

Figure 4: Basic designs of NFT-based transmission systems, including transmission in the NFT domain, DBP with the use of NFT operations, and the hybrid method. The figure was originally published in [3].

First, to solve this problem it was suggested to use the NFT defined for periodic signals instead of originally suggested one for vanishing signals [27–29]. In this case, the transmitter sends only one period of the generated periodical signal and a cyclic prefix, in the same fashion as in conventional optical frequency-division multiplexing (OFDM) systems [30] (Fig. 5). The main problem of the periodic NFT approach is that it is much more mathematically complicated to calculate the NFT for periodic signals [3]. Besides, there is no straight-forward way to define a desired period length. By far, periodic NFT was implemented only for a quite simple systems with low capacity [31,32].

The recently introduced b-modulation NFT technique [25,26,33], operating with the band-limited $b(\xi)$ profiles, has been aimed at resolving one of the principal challenges in the NFT-based communication: to attain the explicit control over the temporal duration of NFT-generated signals at the transmitter side. The latter property allows us to pack our data better inside a given time-bandwidth volume and, thus, to reach higher spectral efficiency numbers. In the case of b-modulation, we map our data on the function $b(\xi)$, which is chosen to be band-limited, and further adjust the function $a(\xi)$ accordingly. Then, the ensuing signal $u(t)$, obtained through the inverse NFT operation, has a finite duration in time domain [33]. The latter feature together with the fact that b-values are less susceptible to noise and inter-carrier interference than $q_c(\xi)$ modulation allows one to get a higher spectral efficiency compared to "conventional" NFT-based systems employing the continuous NF spectrum modulation [3,26].

The most advanced flavour of the b-modulation allowing us to reach highest spectral efficiencies was given in works [25,34]. There, the dual-polarisation NFDM transmission of the b-modulated signal was suggested. Compared to signle polarisation case the transmission of signal here is described by a Manakov equation [35,36] instead of NLS and so b-values form a pair. This works proposed a specific mapping of encoded data to the

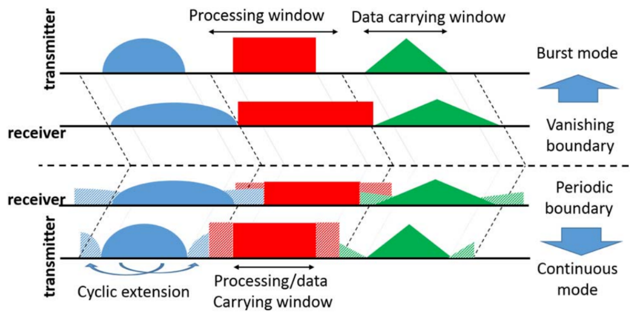

Figure 5: Burst mode for the window in vanishing signal processing (ordinary NFT) and the processing window for the periodic signal with cyclic extension (PNFT). The figure was originally published in [3].

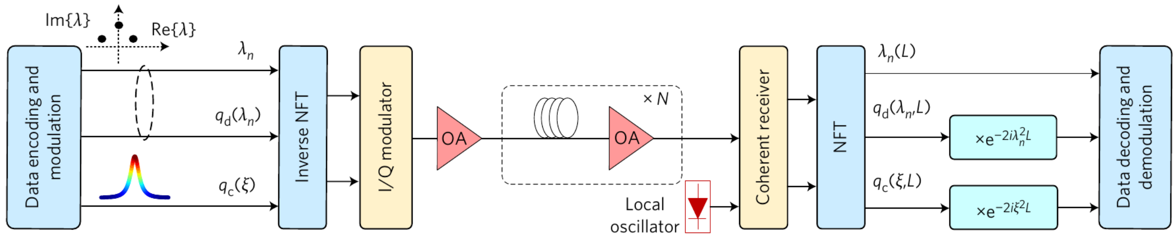

Figure 6: Basic block diagram of NFDM transmissions. There, the transmitted data are encoded onto the nonlinear spectrum and the time-domain signal is generated via the inverse NFT at the transmitter. At the receiver, the interplay between the dispersion and nonlinearity along the link is cancelled by a phase-shift-removal operation in the nonlinear-spectrum domain (for each nonlinear-frequency component). OA, optical amplifier. The figure was originally published in [3].

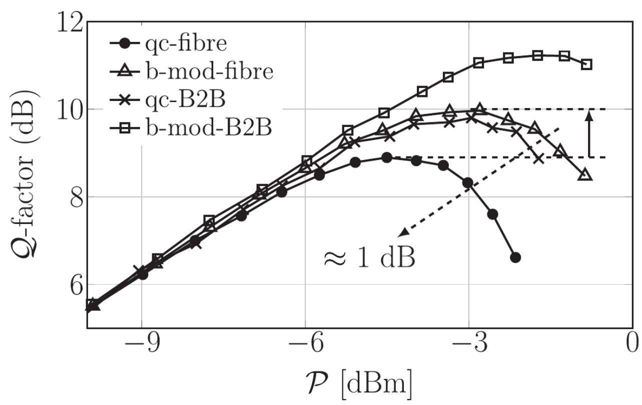

Figure 7: Q-factor vs. signal launch power in q_c- and b-modulated DP-NFDM systems with the guard interval being 4 times longer that the burst extent in both AWGN case and fibre transmission. The total additive noise powers are the same. The figure is taken from [25].

b-coefficient $b(\xi)$

$$\begin{gathered}
\Delta_k = \frac{\sqrt{1-\exp\left(-|u_{1k}|^2-|u_{2k}|^2\right)}}{|u_{1k}|^2+|u_{2k}|^2} \\
b_{1k} = \Delta_k \cdot u_{1k}, \\
b_{2k} = \Delta_k \cdot u_{2k}, \\
a_k = \sqrt{1-|b_{1k}|^2-|b_{2k}|^2} \cdot \exp\left(j\mathcal{H}\left(1/2\log\left(1-|b_{1k}|^2-|b_{2k}|^2\right)\right)\right),
\end{gathered} \tag{2}$$

where $\mathcal{H}$ is the Hilbert transform. Due to this mapping, we cannot longer attain the exact localisation of time domain profiles. Nonetheless, the energy transferred into non-zero wings is relatively small such that the flavour of the b-modulation concept is preserved.

Using such an NFDM system we can efficiently reach 400 Gbit/s data rate. The dependence of the quality factor on the power of transmitted signal compared with the other NFDM systems is given in Fig. 7. So far, it is the highest transmission rate achieved within the NFDM concept. The performance of the system was experimentally demonstrated in [34].

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