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Executive Summary

The distortion induced in the optical channel by Kerr nonlinearity is one of the main bottlenecks of the modern fibre-optic communications. Nonlinear Fourier transform (NFT) is the mathematical technique allowing for cancelling the nonlinear distortion easily. NFT transforms a signal from time domain into a spacial domain, referred to as the nonlinear spectrum, where the complex evolution of a signal by the interplay of Kerr nonlinearity and chromatic dispersion is represented as the linear localized phase shift. In this work, we describe nonlinear frequency division multiplexing (NFDM) systems where the information is encoded into and received from the nonlinear spectrum. Nonlinear inverse synthesis (NIS) systems utilizing only the continuous part of the nonlinear spectrum as an information carrier are a particular case of the broader NFDM concept. We describe NFDM systems utilizing continuous (NIS) and the discrete spectrum. For each type of NFDM system, we bring the main technological breakthroughs and performance milestones achieved.

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LIST OF ACRONYMS

ASE	Amplified spontaneous emission
CD	Chromatic dispersion
CM-NFDM	Continuously modulated nonlinear Fourier division multiplexing
DSP	Digital signal processing
DT	Darboux transform
EC	European Commission
EID	European Industrial Doctorates
ESR	Early Stage Researcher
FONTE	Fibre Optic Nonlinear Technologies
FB	Forward-backward
GLME	Gelfand-Levitan-Marchenko equation
DBP	Digital amplitude modulation
INFT	Inverse nonlinear Fourier transform
NF	Nonlinear Fourier
NFD	Nonlinear Fourier domain
NFDM	Nonlinear Fourier division multiplexing
NFT	Nonlinear Fourier transform
NIS	Nonlinear inverse synthesis
NLM	Nonlinearity mitigation
NLSE	Nonlinear Schrödinger equation
NS	Nonlinear spectrum
OA	Optical amplifier
OFDM	Orthogonal frequency-division multiplexing
QAM	Quadrature amplitude modulation
SMF	Single-mode fiber
ZSSP	Zakharov-Shabat spectral problem

1 Introduction

The optical signal, transmitted over an optical fiber, is distorted because of its interaction with the fiber. Part of this distortion, proportional to the local signal amplitude, is referred to as the fiber nonlinearity. The instantaneous third-order nonlinear response, known as Kerr nonlinearity, constitutes one of the main limits on the performance of fiber optic communications [1]. It corrupts the signal waveform proportionally to the signal intensity, thus introducing the distortions into an information-bearing waveform and coupling signals with optical noise. These distortions make it impossible to mitigate the amplified spontaneous emission (ASE) noise-induced distortions, where the optical noise emerges due to the presence of optical amplifiers (OA), by means of merely increasing the signal power. Therefore, the mitigation of nonlinear interference is believed to be an utterly important issue to improve the overall performance of the system by leveraging higher power levels. A plethora of nonlinearity mitigation (NLM) methods have been proposed up to date [2, 3]. Among them, the digital signal processing (DSP) methods are of the main interest for industry since their implementation doesn't require expensive re-laying of optical fibers [2].

An approach to nonlinearity mitigation is based on the nonlinear Fourier transform (NFT) [4, 5]. NFT term stands for the conversion of the time-domain optical signal to the specific domain, referred to as a nonlinear Fourier domain (NFD) or nonlinear spectral domain. The result of conversion is often named as the nonlinear spectrum (NS) of the signal. The key property of this domain is that the joint effect of the second-order chromatic dispersion (CD) and Kerr nonlinearity on the optical signal can be described as the pointwise singal-independent phase rotation of its NS. This process is similar to the effect of CD on the ordinary Fourier spectrum in a linear channel.

Since NS is immune to the CD and Kerr nonlinearity, it was suggested to encode information by modulating nonlinear spectrum. The information-bearing symbols are first transferred to the time-domain by inverse NFT (INFT). The obtained waveform is propagated through the optical channel. Finally, the received signal is converted back to the NFD by NFT and filtered from CD and nonlinear distortion by applying the spectral mask. The described method is referred to as both nonlinear inverse synthesis (NIS) [6] and nonlinear Fourier division multiplexing (NFDM) [7] (the former term usually refers to our using only the continuous part of NS, i.e. the dispersive nonlinear waves, and borrowing the modulation formats from linear systems). In this article we review the work done in regard to this method and refer to it later as either NIS or NFDM.

2 Nonlinear Fourier transform

Nonlinear Schrödinger equation (NLSE) is the basic model describing the evolution of an optical pulse waveform during pulse propagation over the fiber-optic system

$$iu_z - \frac{\beta_2}{2} u_{\tau\tau} + \gamma |u|^2 u = 0 \tag{1}$$

where z denotes the propagated distance and τ is the time coordinate in the frame moving with the group velocity of the envelope. Here, we consider the case of the constant anomalous chromatic dispersion ($\beta_2 < 0$ in (1)) usual for the modern telecommunications utilising C-band in single-mode fibers (SMFs) [8]. The explicit form of NFT is known for NLSE (1), in other words the equation is integrable. For convenience, NLSE (1) is often non-dimensionalized. The time τ is normalized to the characteristic signal timewidth T_s, the coordinate z - to the effective z-scale derived from T_s: $z_s = T_s^2/|\beta_2|$, the signal amplitude - to $A_s = \sqrt{\gamma Z_s}$. To sum up, the normalization is as follows

$$\tau/T_s \to t, \quad z|\beta_2|/T_s^2 \to y, \quad u\sqrt{|\beta_2|/(\gamma T_s^2)} \to q. \tag{2}$$

The normalized NLSE takes the form

$$iq_y + \frac{1}{2} q_{tt} + |q|^2 q = 0. \tag{3}$$

To calculate the "conventional" NFT of temporal profile $q(y, t)$ by definition requires that

$$\int_{-\infty}^{\infty} dt |q(y,t)| < \infty \text{ for any } y. \tag{4}$$

2.1 Forward NFT

The forward nonlinear Fourier (NF) decomposition of a pulse propagating in the NLSE channel (1) can be performed via the solutions of the so-called Zakharov-Shabat spectral problem (ZSSP) [9] corresponding to the scattering problem for two auxiliary functions $\varphi_{1,2}(t)$, where the transferred temporal profile $q(0,t) \equiv q(t)$ enters as an effective potential:

$$\frac{d}{dt}\begin{pmatrix}\varphi_1(t,\xi)\\ \varphi_2(t,\xi)\end{pmatrix} = \begin{pmatrix}-i\xi & q(t)\\ -q^*(t) & i\xi\end{pmatrix}\begin{pmatrix}\varphi_1(t,\xi)\\ \varphi_2(t,\xi)\end{pmatrix}. \tag{5}$$

where ξ, referred to as the spectral parameter, can be understood as an NFT analogue of frequency and the asterisk stands for the complex conjugation.

Let us consider the particular solution of system (5) called Jost solution $\vec{\Phi} = [\varphi_1(t,\xi), \varphi_2(t,\xi)]^T$. The solution is defined by its initial condition on the left boundary $t \to -\infty$ as

$$\Phi(t,\xi) = \begin{pmatrix}\varphi_1(t,\xi)\\ \varphi_2(t,\xi)\end{pmatrix} \xrightarrow[t\to-\infty]{} \begin{pmatrix}1\\ 0\end{pmatrix}\exp(-i\xi t). \tag{6}$$

Solution (6) has the meaning of the plain wave coming from $t \to -\infty$ to be scattered on the effective potential formed by the transferred signal $q(t)$. For Jost solution $\vec{\Phi}$ the two scattering coefficients $a(\xi)$, $b(\xi)$ can be introduced as

$$a(\xi) = \lim_{t\to\infty}\varphi_1(t,\xi)\exp(i\xi t), \qquad b(\xi) = \lim_{t\to\infty}\varphi_2(t,\xi)\exp(-i\xi t). \tag{7}$$

In other words, Jost solution Φ is expected to behave at $t \to \infty$ as

$$\Phi(t,\xi) = \begin{pmatrix}\varphi_1(t,\xi)\\ \varphi_2(t,\xi)\end{pmatrix} \xrightarrow[t\to\infty]{} \begin{pmatrix}a(\xi)\exp(-i\xi t)\\ b(\xi)\exp(i\xi t)\end{pmatrix}. \tag{8}$$

The scattering coefficeints $a(\xi)$, $b(\xi)$ can also be characterized in terms of a single function, the (right) reflection coefficient

$$r(\xi) = b(\xi)\,/\,a(\xi). \tag{9}$$

Nonliear Spectrum (NS), containing all the information about the original waveform $q(t)$, can be extracted from scattering coefficients (7). NS consists of the two parts:

(i) continuous spectrum, made by $r(\xi)$ for $\xi \in \bar{\mathbb{R}}$;

(ii) discrete spectrum, made by set $\Xi_d \in \mathbb{C}^+$ of solitonic eigenvalues $\xi_d \in \Xi_d$ such as $a(\xi_d) = 0$ along with the residues of the scattering coefficient at the eigenvalues:

$$r(\xi_d) = b(\xi_d)/a'(\xi_d). \tag{10}$$

The expression above is generally valid for the domain of analiticity of $b(\xi)$ [10]. However, in optical communication applications it is always the case as we work with the finite-duration pulses.

The NFT of the signals with the low enough level of energy contains only continuous spectrum while the dicrete part emerges only after an energy threshold is overcome by the temporal profile $\int_{-\infty}^{\infty} dt\,|q(t)|^2 > C$ [11]. In case of normal chromatic dispersion ($\beta_2 > 0$ there is no discrete nonlinear spectrum at all.

The main reason to study NFT in the context of the telecommunications is the simple evolution of the nonlinear spectrum of the signal propagated over the fiber-optic system. For the non-dimensionalized NLSE (3) the evolution of the waveform's NS during its propagation over the link is given as

$$\mathrm{NS}(L) = \begin{cases} b(L,\xi) = b(0,\xi)\exp\bigl(2i\xi^2 L\bigr)\ \forall\xi \in \hat{\mathbb{R}} \cup \Xi_d \\ a(L,\xi) = a(0,\xi), a'(L,\xi) = a'(0,\xi)\ \forall\xi \in \hat{\mathbb{R}} \cup \Xi_d \\ \xi_d(L) = \xi_d(0). \end{cases} \tag{11}$$

2.2 Inverse NFT

The inverse NFT (INFT) converts back the nonlinear spectrum into the time-domain waveform $q(t)$ by means of solving the integral Gelfand-Levitan-Marchenko equation (GLME) for the unknown interim functions $K_{1,2}(\tau,\tau')$ [5,12]. For the anomalous CD ($\beta_2 < 0$) the right nonlinear spectrum, considered there, enters GLME trough the quantity $R(\tau)$ containing contributions from both continuous and discrete (if present) NS parts,

$$R(\tau) = R_c(\tau) + R_d(\tau), \qquad R_c(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\xi\, r(\xi)\, e^{-i\xi\tau}, \qquad R_d(\tau) = i\sum_{\xi_d\in\Xi_d} r(\xi_d)\, e^{-i\xi_d\tau}, \tag{12}$$

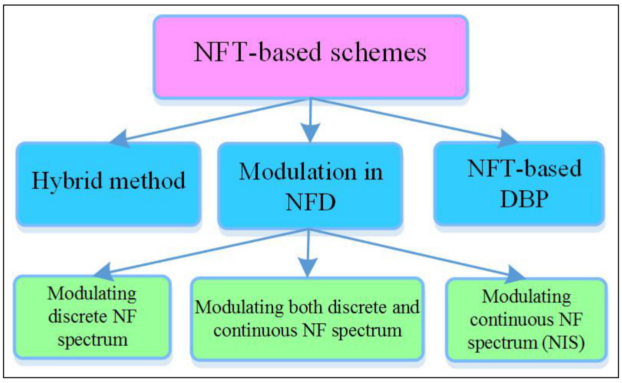

Figure 1: Diagram of the currently proposed and studied NFT-based methods. The figure is kindly provided by the authors of [5].

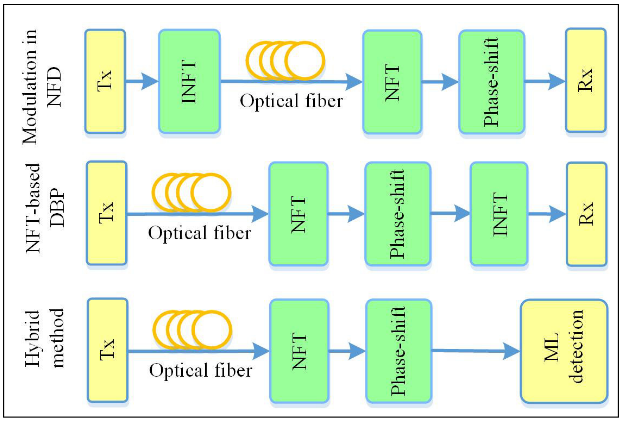

Figure 2: Basic designs of NFT-based transmission systems, including transmission in the NFT domain, DBP with the use of NFT operations, and the hybrid method. The figure is kindly provided by the authors of [5].

where it is assumed that all discrete eigenvalues have a unit multiplicity. In case of normal CD ($\beta_2 > 0$) there is no discrete spectrum contribution and, hence, $R_d(\tau) \equiv 0$ for any localized input. The GLME for the unknown functions $M_{1,2}(\tau, \tau')$ takes the form

$$\begin{aligned} M_1^*(\tau,\tau') + \int_\tau^\infty dy\, R(\tau'+y)\, M_2(\tau,y) = 0\,, \\ -M_2^*(\tau,\tau') + R(\tau+\tau') + \int_\tau^\infty dy\, R(\tau'+y)\, M_1(\tau,y) = 0, \end{aligned} \tag{13}$$

where the asterisk denotes complex conjugation. Having solved GLME (13) for $M_{1,2}(\tau,\tau')$, the temporal profile is eventually recovered via the relation: $q(t) = -2M_1(t,t)$.

3 Principal applications of NFT in fiber-optic communications

Since the evolution of individual NS modes is linear and decoupled, these modes can be effectively employed for coding, transmission, detection and processing of information. So far the three principal designs for NFT-based transmission systems were suggested. They are schematically presented in the Figs. 1, 2. In the first design, the transmitted information is encoded directly onto the NS via the INFT. The design is referred to as "Modulation in the nonlinear Fourier domain (NFD)", which is actually the NFDM. Within this design, the discrete [13] and continuous [6, 14] parts of NS can be modulated separately or together [15]. In the second design, the NFTs are applied to cancel the nonlinear distortion on the receiver or transmitter side. There, the NFTs are applied to calculate the inverse signal propagation similarly to what we have within the traditional digital back-propagation (DBP) [2], while the signal encoding and receiving is done conventionally (e.g, by QAM or OFDM) in the time

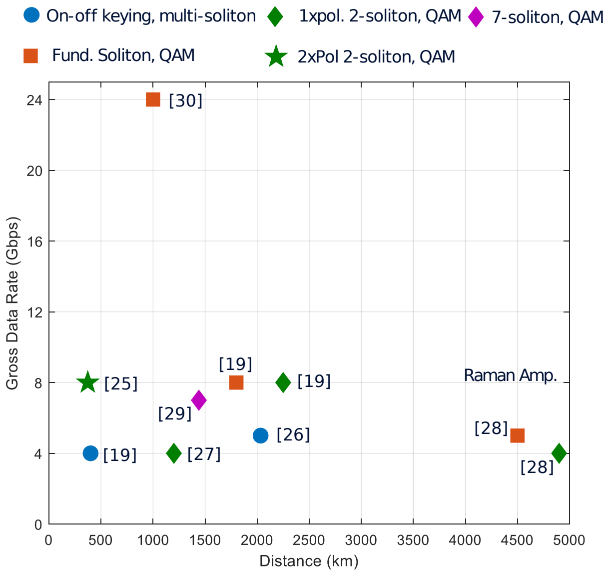

Figure 3: The performance obtained by the pioneering discrete NFDM communication systems. The figure is kingly provided by the author of [20].

domain [16, 17]. Nonetheless, the yet unsolved challenge in the NFT-based DBP is the necessity to calculate precisely the a priori unknown number of discrete eigenvalues in the randomly modulated information-bearing signal. Therefore, the NFT-based DBP approach was developed so far only for the low-energy soliton-free case and for normal dispersion fibres [16,17], where no discrete NS is formed. The third design, named as the "hybrid method", implies information is encoded conventionally in the communication system but the detection involves the NS of the received signal (i.e. [18, 19]).

The further discussion will be limited to the first design, which in effect consitutes the nonlinear inverse synthesis (NIS) paradigm or, more generally, the NFDM concept.

4 NFDM utilising the discrete spectrum modulation

Since the original work by Hasegawa and Nyu [21], the discrete part of nonlinear spectrum (NS), representing stable solitary solutions, was supposed to be the primary target for information coding. Within this approach, first, the transmitted information is coded by a predefined set of discrete eigenvalues Ξ_d and NS parameters related to them: $\xi_d \in \Xi_d, b(\xi_d), a'(\xi_d)$. Next, the corresponding time-domain waveforms are generated using the INFT. At the receiver, NFT is applied to recover modulated discrete eigenvalues and norming constants.

Several key technologies significantly improved the transmission quality of the systems utilising discrete NS. First, the application of Darboux transformation significantly eased the application of INFT to a discrete multi-eigenvalue spectrum. The Darboux transform (DT) describes analytically the changes in a temporal profile and its nonlinear spectrum caused by adding a new eigenvalue, i.e. $\{\xi_d, b(\xi_d), a'(\xi_d)\}$, to the nonlinear spectrum. DT was first used to compose temporal profiles corresponding to the multi-eigenvalue nonlinear spectrum in [22]. Later, the method of the application of DT to the construction of multi-eigenvalue solution was further simplified in [23]. The fast DT numerical algorithm with the nearly linear computational cost was proposed in [24]. The second important advancement in the discrete NS part exploitation is the forward-backward (FB) algorithm [23, 25] for the retrieval of spectral amplitudes, which allowed us to compute the discrete spectral amplitudes with more precision and to restore accurate enough the phase and amplitude information of discrete spectral components $\arg(b(\xi_d))$. The method uses the fact that scattering coefficient $b(\xi_d)$, in addition to its definition (7), can be introduced via the relation between the two Jost solutions, i.e. through the waves incident from $t \to -\infty$ and $t \to \infty$. Hence, numerical estimations of both these solutions can be used to find the coefficient $b(\xi_d)$, instead of naive single Jost solution-based method (8). Third, it was shown in [19] that the spectral coefficient $b(\xi_d)$ contains all the information regarding amplitude and phase of the discrete NS component $r(\xi_d) = b(\xi_d)/a'(\xi_d)$, and so it can be used during detection instead of $r(\xi_d)$. This allows us to exclude the noise injected into $a'(\xi_d)$ from the detection procedure because of the divergence of real fiber-optic channel from the ideal NLSE model (1); we shall call this noise as a divergence noise. Furthermore, in [19] it was shown that the divergence noise entering into the detected value of $b(\xi_d)$, is correlated with the one appearing into detected ξ_d and $a'(\xi_d)$ values, such

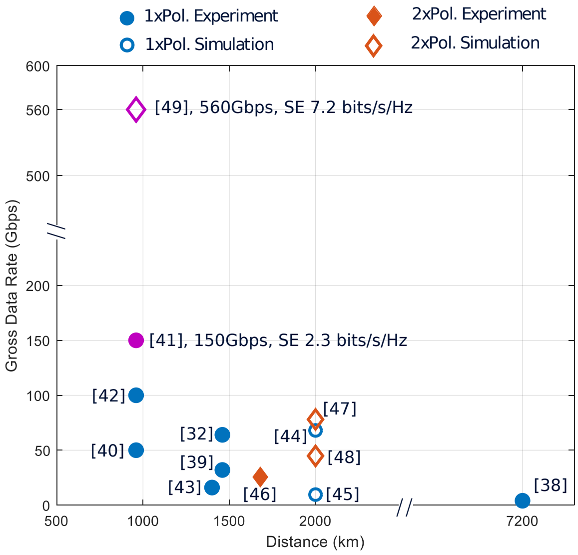

Figure 4: The performance obtained by the pioneering continuous and dual continuous-and-discrete NFDM communication systems. The figure is kingly provided by the author of [20].

that the latter ones can be used to decrease the noise contribution in $b(\xi_d)$. Finally, the extension of originally single-polarization NFT for the two-polarization case is suggested in [26].

The aforementioned technologies allowed the several performance milestones to be achieved in different systems. The performance, achieved by different pioneering discrete spectrum systems [19, 26–31] is compared in Fig. 3.

5 NFDM utilising both discrete and continuous spectrum modulation

Originally, in view of [21] the NFDM was supposed to operate only with the discrete parts of nonlinear spectrum. First NFDM scheme based on information encoding onto the continuous part $\xi \in \mathbb{R}^+$ of the nonlinear Fourier spectrum has been proposed in [6]. Despite continuous spectra typically allows one to encode more information than the discrete one (like in [32]), it has a particular problem non-present in a discrete NS: the width in time-domain of a continuously modulated NFDM (CM-NFDM) signal increases during propagation because of dispersive broadening. To prevent inter-pulse interference during their propagation, zero guard intervals should be introudced between the adjacent CM-NFDM-modulated pulses in the time-domain. The duration of the intervals should be longer than the channel memory, induced by chromatic dispersion. The necessity to have guard intervals undercuts the spectral efficiency of the CM-NFDM-based communication systems. Therefore, the main achievements in CM-NFDM were obtained by shortening either the timewidth of the pulses or the guard intervals between them. First, the work [33] showed that the length of the required guard interval can be decreased in two times if one-half of the transmission-induced phase shift of the NS is pre-compensated at the transmitter and the other half is compensated at the receiver. Second, it was shown in [34, 35] that by applying the particular modulation to spectral coefficient $b(\xi)$ instead of $r(\xi) = b(\xi)/a(\xi)$ it is possible to limit the width of CM-NFDM pulses. Particularly, it was found that the temporal width of the pulse is twice lower than the width of the conventional inverse Fourier transform of its $b(\xi)$ spectral coefficient. This method i sreferred to as the b-modulation. Finally, the b-modulated CM-NFDM system was expanded for the case of two polarizations in [36], which yielded the significant rise in spectral efficiency.

The modern systems utilising the modulation of continuous spectrum typically include the modulation of discrete spectrum too. The simplified Darboux transform originally suggested in [23] for performing numerically the INFT of multi-eigenvalue discrete spectrum was shown in [37] to be applicable for adding a multi-eigenvalue discrete spectrum to the INFT taken from a purely continuous nonlinear spectrum. This allowed the creation of combined continuous-and-discrete modulation schemes, which utilise all available degrees of freedom inside the nonlinear spectrum for the modulation.

Several performance achievements [32, 38–49] reached by various communication systems modulating continuous NS with or without the discrete NS are given in Fig. 4.

6 Conclusion

This deliverable reviewed various approaches to nonlinear frequency-division multiplexing (NFDM) - encoding information by modulating the nonlinear spectrum. The nonlinear spectrum consists of the two distinct parts - continuous and discrete, each of which requires specific technologies to be used effectively. Here, for both continuous and discrete spectrum-based NFDM we separately mentioned the most important pieces of these technologies and highlighted landmark results achieved by their application. Echoing the findings of the authors of [5] published in 2017, we conclude that NFDM systems haven't yet reached the performance level of the state-of-the art fibre-optic communication systems. Nonetheless, NFDM systems have shown considerable improvement since the publication of [5]: the b-modulation [34] was introduced in continuous spectrum modulation and dual-polarisation modulation has been suggested for both continuous [36, 49] and discrete [26] nonlinear spectrum. We suggest that the further development of NFDM systems most probably would be caused by the application of advanced optimisation techniques, including machine learning approaches, to the tuning the parameters of NFDM systems.

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