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Executive Summary

Wavelength-division multiplexing (WDM) and nonlinear frequency-division multiplexing (NFDM) are the two multiplexing schemes for optical fiber communication. In WDM, which is the same as linear frequency-division multiplexing (FDM) in radio communication systems, user's signals are linearly multiplexed in the frequency domain. However, in nonlinear channels, such as optical fibers, linear multiplexing causes interactions. To address this, NFDM has been proposed. In NFDM, which is based on the nonlinear Fourier transform (NFT), users' signals are multiplexed in the nonlinear Fourier domain and propagate independently in a lossless noiseless optical fiber modeled by the nonlinear Schrödinger (NLS) equation.

In light of recent notable progress in these schemes, ever-increasing attention has been attracted to this area. In this report, mathematical principles underlying modulation and multiplexing in linear and nonlinear communication systems are reviewed.

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LIST OF ACRONYMS

AiPT	Aston Institute of Photonic Technologies
AIR	Achievable Information Rate
AWGN	Additive White Gaussian Noise
DoFs	Degrees-of-Freedoms
EC	European Commission
EID	European Industrial Doctorates
ESR	Early Stage Researcher
FONTE	Fibre Optic Nonlinear Technologies
ICI	Inter-Channel Interference
ISI	Inter-Symbol Interference
NFDM	Nonlinear Frequency-Division Multiplexing
NFT	Nonlinear Fourier Transform
NLS	Nonlinear Schrödinger
ODFM	Orthogonal frequency-division multiplexing
SSFM	Split-Step Fourier Method
WDM	Wavelength-Division Multiplexing

1. MATHEMATICAL PRELIMINARIES

Communication systems are designed based on mathematical principles. In this section, a number of mathematical theorems underlying the principles of modulation and multiplexing for communication systems are discussed.

Definition 1.1. An inner product on a vector space E over a field K is a function $<.,.>: E \times E \rightarrow K$ satisfying the following properties [1]:

a) $\forall\, x, y \in E, < x, y > = \overline{< y, x >}$;
b) $\forall\, x, y \in E, \forall\ \lambda \in K \quad < \lambda_1 x_1 + \lambda_2 x_2 , y > = \lambda_1 < x_1, y > + \lambda_2 < x_2, y >$;
c) $\forall x \in E, < x, x > \geq 0 \ \ \& < x, x > = 0 \ \text{ iff. } \ x = 0.$

An inner product space is a vector space together with an inner product. ■

Definition 1.2. A Hilbert space H is a real or complex inner product space that is also a complete metric space with respect to the distance function induced by the inner product [1]. ■

Theoream 1.1. Every Hilbert space has an orthonormal basis [1]. ■

Theoream 1.2. The set of all finite energy functions forms a Hilbert space over field $\mathbb{C}$ with the following inner product [1]:

$$< f(t), g(t) > = \int_{-\infty}^{+\infty} f(t)\overline{g(t)}dt\,. \tag{1}$$ ■

Therefore, every finite-energy function f in $\mathbb{C}$ can be decomposed into an orthonormal basis $\{\varphi_1, \ \varphi_2, \ldots\}$ of the space as the following:

$$f(t) = \sum_k s_k \varphi_k\,(t)\,, \tag{2}$$

where $s_k \in \ \mathbb{C}$ are scalars.

For the given Hilbert space by (1), the following set serves as orthonormal basis:

$$\{sinc(t - kT_0)\}_{k=-\infty}^{+\infty}, \quad T_0 = \frac{1}{B}\,. \tag{3}$$

However, since the interval in (1) is $[-\infty, +\infty]$, if such an operation is applied in practice, it would be too time consuming. To address this, given in communication, signals are compactly supported, the Hilbert space H_T of signals that are periodic with period T and the following inner product is considered [2]:

$$< f(t), g(t) > = \tfrac{1}{T}\int_0^T f(t)\overline{g(t)}dt\,. \tag{4}$$

An orthonormal basis of H_T is

$$\{\cos(\frac{2k\pi}{T}t), \sin(\frac{2k\pi}{T}t)\}_{k=1}^{+\infty}\,. \tag{5}$$

Modulation is the process of mapping symbols s_k to function $f(t)$. This can be done by expanding a signal in the orthonormal basis of the space, according to (2). The merit of this approach is enabling the receiver to decode the transmitted symbols simply by projection of the received signal into the orthonormal basis (demodulation). In an identity channel $Y = X$, this can easily be done as follows.

Let the signal $X(t) = \sum_{k=1}^{n} s_k \varphi_k(t)$ be transmitted. The receiver can easily demodulate the symbols s_i as follows:

$$\begin{aligned} \widehat{s_i} &= < Y, \varphi_i > \\ &= < X, \varphi_i > \\ &= < \sum_{k=1}^{n} s_k \varphi_k \,, \varphi_i > \\ &= \sum_{k=1}^{n} s_k < \varphi_k, \varphi_i > \\ &= s_i \,. \end{aligned} \tag{6}$$

The schematic diagram of a communication system for the single-user case is illustrated in Fig. 1.

Multiplexing is similar to modulation, but for combining user's signals in multi-user channels. In multiplexing, the goal is to combine users' signals in a way that at the receiver, they can be straightforwardly decomposed (demultiplexing). This can easily be done by choosing the basis supported on disjoint frequency bands of the channel and combining the signals X_i of each user i as $\hat{X}(f) = \sum_{i=1}^{n} \widehat{X_i}(f)$, where $\hat{X}(f)$ is the Fourier transform of $X(t)$.

However, communication channels in real-world are not identity channels. They are subject to noise and distortion, depending on the modulation and multiplexing scheme. In the next section, we will review these distortions within discussing transmission over linear channels.

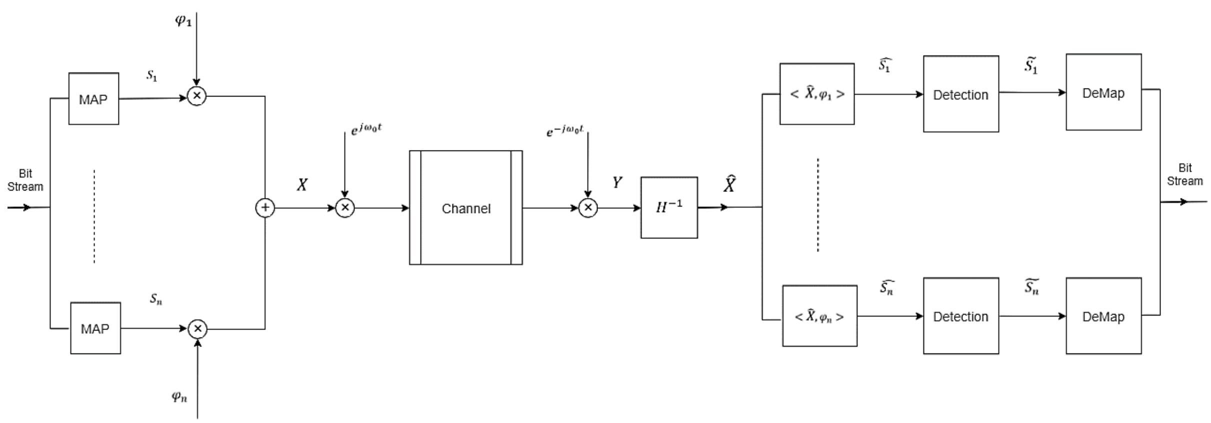

Fig. 1. The schematic diagram of a communication system for the single-user case

2. TRANSMISSION OVER LINEAR CHANNELS

As mentioned in the previous section, a channel, depending on the adopted modulation and multiplexing scheme, may cause distortion in the signal. For instance, consider the canonical communication channel, the additive white Gaussian noise (AWGN) channel:

$$Y(t) = X(t) * H(t) + N(t), \tag{7}$$

where X is the input signal, Y is the output signal, H is the channel filter, N is the white Gaussian noise, and $*$ denotes convolution. Using (2) and (6) we obtain

$$\begin{aligned} y_i &=< \left(\sum_k x_k \varphi_k(t) * \ H(t) \right) + \ N(t), \varphi_i(t) > \\ &= \sum_k x_k < \varphi_k(t) * H(T), \varphi_i(t) > + < N(t), \varphi_i(t) > , \end{aligned} \tag{8}$$

which results in:

$$y_i = \sum_k x_k h_{ik} + n_i = x_i h_i + \overbrace{\sum_{k \neq i} x_k h_{ik}}^{Interference} + \overbrace{n_i}^{Noise} , \tag{9}$$

where $h_i = h_{ii}$, and $h_{ik} =< \varphi_k(t) * \ H(t), \varphi_i(t) >$. If x_ks represent symbols of the user-of-interest, the interference in (9) is called **inter-symbol interference (ISI)**, and if they represent symbols of other users, it is called **inter-channel interference (ICI)**. As (9) depicts, three distortions are occurred, which can be categorized into two groups: deterministic and stochastic [5]. In the following, these distortions are conferred.

2.1 Stochastic Distortion

Almost always there is noise in the channel. Noise is modeled by a probability distribution. As a result, they change the signal in a way that the exact final output cannot be determined prior to arrival, but only a prediction of it. For example, the thermal noise is the primary noise source in many channels which is modeled by a white Gaussian random process.

Owing to the probabilistic nature of this process, which is represented by n_i in (9), this distortion is called stochastic distortion.

2.2 DETERMINISTIC DISTORTION

In the absence of noise, given the mathematical model of the channel, the evolution of a signal in the channel can be calculated deterministically. For example, the evolution of signals in a lossless noiseless optical fiber obeys non-linear Schrödinger (NLS) equation [2]. From this point of view, the change that a channel, in the absence of noise, imposes on an input signal is called deterministic distortion.

Deterministic distortion is divided into two parts. One part is a function of the symbol-of-interest/user's signal, and the other is interference.

2.3 Interference

As (9) depicts, the interference causes y_k to be dependent on all x_is rather than only x_k, which makes the process of demultiplexing/demodulation more complicated.

In a multi-user channel, given the multiplexing scheme, e.g., linear multiplexing $X = \sum_l (\sum_k x_k \varphi_k) \varphi'_l$, both inter-symbol interference and inter-channel interference can occur.

This matter will be highlighted in the subsequent section, describing a couple of multiplexing schemes for nonlinear communication systems, such as optical fibers.

3. TRANSMISSION OVER NONLINEAR CHANNELS

Based on the potential distortions that may occur in linear channels, effective linear schemes have been proposed for these channels. However, in nonlinear channels, such as optical fibers (which is modeled by nonlinear Schrödinger equation [2]), these linear schemes dramatically degrade the performance. Towards overcoming these issues, NFDM has been proposed, which is based on the nonlinear Fourier transform (NFT). In this section, the principles of these linear and nonlinear approaches in fiber-optic communication are reviewed.

4.1 LINEAR FREQUENCY-DIVISION MULTIPLEXING IN OPTICAL FIBER

Linear multiplexing in fiber-optics is represented by the wavelength-division multiplexing scheme. As the name indicates, wavelength-division multiplexing is the method of multiplexing disjoint wavelengths onto the same fiber. Conceptually, WDM scheme is the same as frequency-division multiplexing (FDM) for radio communication systems [6].

The transmitted signal over the channel in a WDM optical system is as follows [7]:

$$q(t, z=0) = \sum_{K=1}^{N} \left(\sum_{l=1}^{WT} s_k^l \varphi_l(t) \right) e^{jk2\pi Wt}, \qquad 0 \le t \le T, \tag{10}$$

where $q(t,z): \mathbb{R} \times \mathbb{R} \to \mathbb{C}$ is the complex envelope of the signal as a function of time t and distance z along the fiber (the transmitter is located at $z = 0$), $\varphi_l(t)$ are the basis in the time, s_k are the transmitted symbols, $W \ge 1/T$ is the bandwidth per channel, and N is the number of WDM channels. As a particular case, consider each user send an isolated pulse in the time interval $[0, T]$. In this case, a single frequency is transmitted by each user in a bandwidth $W = 1/T$ and $q(t, 0) = \sum_1^N q_k(0) \exp(\frac{jk2\pi t}{T})$, where $\{q_k(0)\}$ are the Fourier series coefficients at $z = 0$. The evolution of Fourier series of this signal is

$$q(t, z) = \sum_{K=1}^{N} q_k(z) \mathrm{e}^{jk\omega_0 t}, \qquad \omega_0 = \frac{2\pi}{T}. \tag{11}$$

The propagation of a signal in a lossless noiseless optical fiber can be modeled by the nonlinear Schrödinger (NLS) equation [2]:

$$j\frac{\partial q}{\partial z} = \frac{\partial^2 q}{\partial t^2} + 2|q|^2 q + n(t, z), \tag{12}$$

where $n(t, z)$ is (zero-mean) white circular symmetric complex Gaussian noise.

By substituting the periodic solution (11) into (12), the NLS equation in the discrete frequency domain is achieved [7]:

$$j\frac{\partial q_k(z)}{\partial z} = -{\omega_0}^2 k^2 q_k(z) + \overbrace{F_k\big(q_1(z), \dots, q_N(z)\big)}^{Interference} + \overbrace{n_k(z)}^{noise}, \tag{13}$$

where

$$F_k = \overbrace{2|q_k(z)|^2 q_k(z)}^{SPM} + \underbrace{2q_k(z)\sum_{1\neq k}|q_l(z)|^2}_{XPM} + \underbrace{2\sum_{m\neq k\ \&\ l\neq k} q_m(z)q_l(z)\,q^*_{m+l-k}(z)}_{FWM}, \quad (14)$$

and $n_k(z)$ is the noise coordinate in frequency. Therefore, as (13) and (14) illustrates, the signal evolution includes dispersion, self-phase modulation (SPM), cross-phase modulation (XPM), and four-wave mixing (FWM) terms in the frequency domain.

Despite all the advantages of WDM, these distortions diminish the performance of WDM dramatically. Towards overcoming these problems to achieve a higher achievable information rate (AIR), nonlinear frequency-division multiplexing scheme has been proposed. This scheme has been shown to have a higher AIR for a given bandwidth and average signal power. In the following part, this scheme will be briefly reviewed.

4.2 NONLINEAR FREQUENCY-DIVISION MULTIPLEXING

In this section, the principles of nonlinear frequency-division multiplexing is briefly reviewed [8].

Consider the channel:

$$Y = R(X) + N, \quad (15)$$

where $R: \mathcal{H} \to \mathcal{H}$ is a compact map on a separable complex Hilbert space $\mathcal{H}$ with the inner product $<,>$; X is the input signal, Y is the output signal and N is the Gaussian noise on $\mathcal{H}$. By projecting the signals and noise onto an orthonormal basis $(\varphi_\lambda)_{\lambda\in\mathbb{N}}$ of $\mathcal{H}$, the channel can be discretized as follows:

$$\{X, Y, N\} = \sum_{\lambda=1}^{\infty}\{X_\lambda, Y_\lambda, N_\lambda\}\varphi_\lambda, \quad (16)$$

where $X_\lambda, Y_\lambda, N_\lambda \in \mathbb{C}$ are the degrees-of-freedoms (DoFs). This leads to obtaining the following discrete model:

$$Y_\lambda = H_{\lambda\lambda}X_\lambda + \overbrace{\sum_{\mu\neq\lambda} H_{\lambda\mu}X_\mu}^{linear\ interaction} + N_\lambda, \quad (17)$$

where $H_{\lambda\mu} = < R\varphi_\mu, \varphi_\lambda >, \lambda \in \mathbb{N}$. Contingent on the basis, interactions in (17) can represent ISI in time or ICI in frequency.

Let R be diagonalizable and have a set of eigenvectors which forms an orthonormal basis of $\mathcal{H}$. (e.g., the case where R is self-adjoint). By this basis, interactions in (17) become zero and therefore it turns to

$$Y_\lambda = H_\lambda X_\lambda + N_\lambda, \quad (18)$$

where $H_\lambda = H_{\lambda\lambda}$ is an eigenvalue of R . This causes the channel to be decomposed into parallel independent scalar channels for $= 1, 2, \ldots$.

The origin of the interactions in (17) is incompatibility between the used basis for the communication and the channel. To address this, orthogonal frequency-division multiplexing (OFDM) has been proposed. In OFDM, information is modulated in independent spectral amplitudes X_λ in order to eliminate ISI and ICI as follows.

Let $\mathcal{H} = L_p^2([0, \Gamma])$ and R be the convolution map $R(X) = H(t) * X(t)$. The eigenvectors and eigenvalues of R are

$$\varphi_\lambda(t) = \frac{1}{\sqrt{\Gamma}} e^{-j\lambda\omega_0 t}, \qquad \omega_0 = \frac{2\pi}{\Gamma}, \tag{19}$$

and $H_\lambda = \mathcal{F}(H)(\lambda\omega_0)$. In this way, convolution is mapped to a multiplication operator, as in (18), by using the Fourier transform, in which X_λ, Y_λ and N_λ are Fourier series coefficients. It can be seen that ISI and ICI do not exist in the Fourier basis.

NFT and NFDM are reviewed in [8] in analogy with OFDM. Let $Q(\lambda, z)$ be the NFT of $q(t, z)$ with respect to t. As a property of NFT,

$$Q(\lambda, \mathcal{L}) = H(\lambda, \mathcal{L}) Q(\lambda, 0), \tag{20}$$

if $q(t, z)$ propagates in NLS (12) with the noise set to zero [8]. In (20), $\mathcal{L}$ is the distance where the receiver is located, and $H(\lambda, \mathcal{L}) = \exp(-j4\lambda^2\mathcal{L})$. Therefore, similar to the Fourier transform, which converts a linear convolutional channel into a number of parallel independent channels in frequency, NFT converts the noiseless nonlinear dispersive channel (modeled by NLS), into a number of parallel independent channels in nonlinear frequency. In NFDM, information is modulated in independent spectral amplitudes $Q(\lambda)$ for every λ. In consequence, ISI and ICI become simultaneously zero for all users of a noiseless multi-user channel. As a result, as opposed to WDM, the NFDM AIR is infinite in the deterministic model at any non-zero power. In addition, in practice, it has shown that NFDM has a higher AIR than WDM for a given power and bandwidth, in an integrable model of the optical fiber in the defocusing regime [8]. The comparison between these schemes presented by [8] is illustrated in Fig. 2.

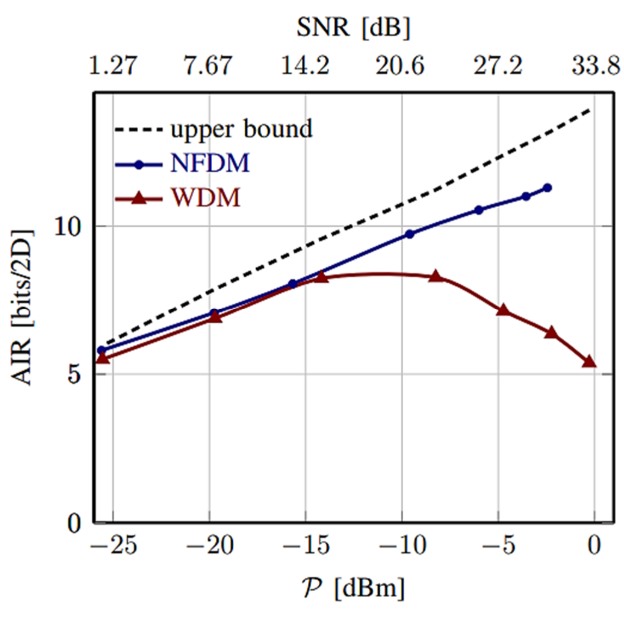

Fig. 2. Comparison between the AIRs of NFDM and WDM, and the capacity upper bound [8].

4. CONCLUSION

After presenting the mathematical basis of modulation and multiplexing in communication systems, in this report, we reviewed the principles of linear modulation and multiplexing in linear and nonlinear channels alongside their corresponding distortions. We showed that WDM, despite some advantages, suffers from inter-channel nonlinear interactions, which significantly limit its performance. The Interactions from many of which, NFDM do not suffer.

APPENDIX: SPLIT-STEP FOURIER METHOD (SSFM)

As discussed, the signal evolution in optical fibers can be modeled by the NLS equation. It has been shown that NLS can be computed by the split-step Fourier method [9]. Briefly describing, in SSFM, both space and time are discretized into $\{z_0, z_1, \dots z_K\}$, $\{t_0, t_1, \dots, t_{L-1}\}$, respectively (the values in both sets are uniformly-spaced, i.e., $z_k = \Delta_z k$ and $t_l = \Delta_t l$), and the evolution of $\underline{a}(z)$ from the position $z = 0$ to position $z = z^*$ is computed recursively as follows [9]:

$$\underline{a}(z_{k+1}) = \mathcal{F}^{-1} D_L \mathcal{F} D_N \, \underline{a}(z_k) + n(z_{k+1}), \tag{21}$$

where $\mathcal{F}$ is the Fourier transform, $\underline{a}(z_k)$ is the $L \times 1$ length-vector of sample values $a(z_k, t_l), l = 0,1, \dots, L - 1$, at position z_k, and n is the Gaussian noise $L \times 1$ length-column-vector. D_N is a diagonal matrix with entries:

$$e^{j\gamma |a(z_k,t_l)|^2 \Delta z}, l = 0,1, \dots, L - 1, \tag{22}$$

where γ is the nonlinearity parameter. D_L is a diagonal matrix with entries:

$$e^{-j\left(\frac{\beta_2}{2}\right) l^2/(L\Delta t)^2 \Delta z}, l = 0,1, \dots, \frac{L}{2} - 1, \tag{23}$$

$$e^{-j\left(\frac{\beta_2}{2}\right)(L-l)^2/(L\Delta t)^2 \Delta z}, l = \frac{L}{2}, \dots, L - 1, \tag{24}$$

where β_2 is the dispersion parameter.

During the previous weeks, we implemented SSFM in Python.

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