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Executive Summary

D2.2 Software implementations of the developed robust NFT algorithms

The proposal of using nonlinear Fourier transform (NFT) for data transmission through optical fibers [1-3] has triggered extensive research in the development of algorithms for the fast computation of NFTs for fiber optics [4-6].

NFT based transmission techniques however are still far from practical implementation due to the major challenges which come from loss and noise. In order to overcome the challenges imposed by fiber loss, we have recently investigated the use of a suitably tapered fiber for NFDM systems [7]. Such fibers are specially designed to overcome the challenge of signal power attenuation and make the NFDM transmission exact. The (I)NFT operations at both transceiver sides need to be adapted for such fibers. Furthermore, specialized code is required in order to simulate transmissions. In this report, we describe our implementations of both the specialized (I)NFT and fiber simulation algorithms. They have both been added to NFDMLab, which is a publically available open source simulation environment for fiber-optic transmission systems based on NFTs, together with specific simulation code that can recreate the examples presentation in our paper [7].

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LIST OF ACRONYMS

AiPT	Aston Institute Of Photonic Technologies
DDF	Dispersion Decreasing Fiber
EC	European Commission
EID	European Industrial Doctorates
ESR	Early Stage Researcher
FONTE	Fibre Optic Nonlinear Technologies
NFT	Nonlinear Fourier Transform
NFDM	Nonlinear Frequency Division Multiplexing
NLSE	Non-Linear Schrodinger Equation

1 INTRODUCTION

The nonlinear Fourier transform can solve certain classes of nonlinear evolution equation [10]. An example of such evolution equation is nonlinear Schrödinger equation (NLSE) which governs the propagation of optical pulse in an optical fiber. The NFT enables the recovery of received signals from transmitted signals by simple phase rotations. This property of simplifying the signal evolution attracted much research on the application of NFTs in data transmission through optical fiber. Open source implementations of fast algorithms for the computation of NFT and INFT have already been publically available in the online GitHub repository "FNFT" [8]. Furthermore, an open source simulation environment called NFDMLab, which builds on FNFT and simulates NFDM based point to point optical communication link, has already been available [9]. NFDMLab contains examples of NFDM based transmission systems demonstrated through experiments. NFDM is originally suitable only for transmission in lossless fiber-optic link, hence, for the case of transmission in lossy fibers the path average model of the NLSE is proposed [11,12]. This method is an approximation, hence, it adds a penalty to the performance of the NFDM. The existing examples in the software library NFDMLab have been using this model. The performance degradation associated with the path average model can be avoided by using dispersion decreasing fiber (DDF). By using dispersion decreasing fiber the approximation can be removed and NFDM transmission made exact. The theory and software implementation of a NFDM system with DDF are described in the next section.

2 EXACT NFDM: THEORY AND SOFTWARE IMPLEMENTATION

The propagation of the slowly-varying complex optical field envelope $Q(\ell,t)$ in a single mode fiber (SMF) can be modelled quite accurately by the NLSE [11, Ch. 2.6.2].

$$\frac{\partial Q}{\partial \ell}+i\frac{\beta_2}{2}\frac{\partial^2 Q}{\partial t^2}-i\gamma|Q|^2Q=-\frac{\alpha}{2}Q \tag{1}$$

where ℓ represents the propagation distance and t is retarded time. The parameters α, β_2 and γ are the loss, dispersion and nonlinear coefficients respectively. Here, we consider the anomalous dispersion case $\beta_2 < 0$. For the case of lossy propagation i.e. $\alpha \neq 0$, the above equation cannot be solved using NFT. This challenge can be addressed by using DDF. DDFs were proposed in classical soliton systems to overcome the broadening of soliton due to loss in optical fiber [13-15]. The dispersion profile of such fibers is tailored along its length in such a way that the nonlinear effects are counteracted by dispersive effects. The propagation of a single soliton in such fiber is shown in figure 1. Inspired by the classical soliton systems, we proposed to use a DDF for NFDM systems in order to avoid the performance degradation induced by the path-averaged method. A practical method to achieve a decreasing dispersion profile is by controlling the diameter of the optical fiber. The diameter of optical fiber can be controlled changing the speed of winding drum during the draw process as shown in figure 2 [11, Ch. 9.3.1]. We assume a simplified approximate relation between the effective core radius r and the dispersion parameter β_2 that was given in [13],

$$r(\beta_2)=\left(\frac{\beta_2}{\kappa}+20\right)/8 \text{ in } \mu\text{m}; \tag{2}$$

where $\kappa = \lambda_0^2 \times 10^{-6}/2\pi c$ and λ_0 and c are the wavelength and speed of light in free space respectively. The nonlinear parameter depends on the effective core radius as follows [11, Ch. 2.6.2]

$$\gamma=\frac{2\pi n_2}{\pi r^2}, \tag{3}$$

where n_2 is nonlinear-index coefficient.

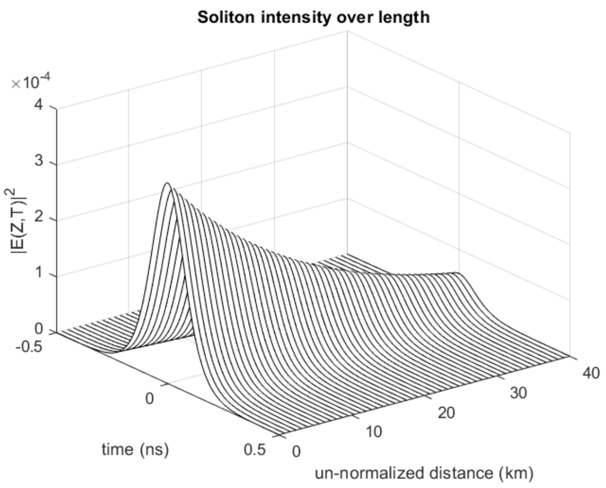

Figure 1: [Soliton propagation in a dispersion decreasing fiber]

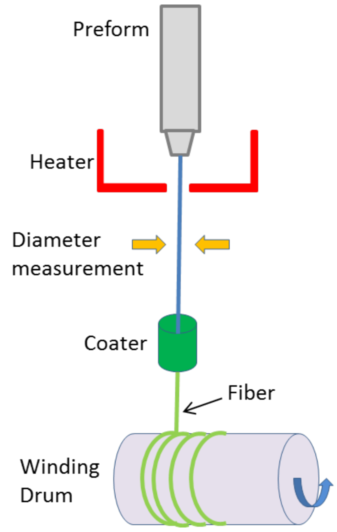

Figure 2: [Fiber drawing process]

As a result, the fiber will have a variable dispersion parameter $\beta_2(\ell) = \beta_2(0)D(\ell)$ and a variable nonlinear parameter $\gamma(\ell) = \gamma(0)R(\ell)$. The propagation of the complex envelope of field $Q(\ell, t)$ in such a fiber is then given by [8],

$$\frac{\partial Q}{\partial \ell} + i\frac{\beta_2(0)D(\ell)}{2}\frac{\partial^2 Q}{\partial t^2} - i\gamma(0)R(\ell)|Q|^2 Q = -\frac{\alpha}{2}Q. \tag{4}$$

By a change of variables $q = Q/\sqrt{P}$, $z = \ell/L_D$, $\tau = t/T_0$ where $L_D = T_0^2/\beta_2(0)$ and $P = 1/\gamma L_D$ and T_0 is a free parameter, the above equation can be transformed into the normalized form [16],

$$\frac{\partial q}{\partial z} + i\frac{D(z)}{2}\frac{\partial^2 q}{\partial \tau^2} - iR(z)|q|^2 q = -\frac{\alpha L_D}{2}q, \qquad q = q(z, \tau). \tag{5}$$

It was shown in [16] that the above equation can be solved exactly via NFT if

$$\alpha L_D = -\frac{\left[R(z)D'^{(z)} - R'(z)D(z)\right]}{R(z)D(z)}, \tag{6}$$

where the prime denotes differentiation. In order to satisfy the above condition, the required dispersion profile has to satisfy

$$\frac{\beta_2(\ell)}{\gamma(\ell)} = \mathrm{A}\, e^{-\alpha\ell}, \tag{7}$$

where $A = \beta_2(0)/\gamma(0)$. By combining (2), (3) and (7), we arrive at equation

$$8\, r^3(\ell)\kappa - 20\kappa\, r^2(\ell) - 2\,\frac{n_2}{\lambda}Ae^{-\alpha l} = 0. \tag{8}$$

The real-valued solution of the above equations gives the effective core radius, from which one can find $\beta_2(\ell)$ and $\gamma(\ell)$. The parameters of 80 km of DDF for $\alpha = 0.2$ dB/km are shown in figure 3.

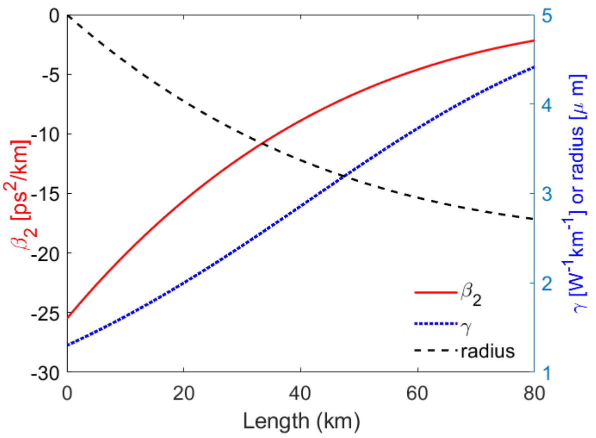

Figure 3: [Parameters of dispersion decreasing fiber of 80 km length.]

Next, we introduce an appropriate NFT subject to the condition. For the standard NLSE that is used normally,

$$\frac{\partial u}{\partial z} + i\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} - i|u|^2 u = 0, \qquad u = u(z,\tau), \tag{9}$$

the forward NFT requires the solution of the so-called Zakhrov-Shabat scattering problem [17].

$$\frac{\partial}{\partial \tau}\begin{pmatrix} v_1(z,\tau) \\ v_2(z,\tau) \end{pmatrix} = \begin{pmatrix} -j\lambda & u(z,\tau) \\ -u^*(z,\tau) & j\lambda \end{pmatrix}\begin{pmatrix} v_1(z,\tau) \\ v_2(z,\tau) \end{pmatrix}, \tag{10}$$

with the boundary condition

$$\begin{pmatrix} v_1(z,\tau) \\ v_2(z,\tau) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-j\lambda\tau} \quad \text{for } \tau \to -\infty. \tag{11}$$

The Jost scattering coefficients are defined as

$$a(\lambda,z) = \lim_{\tau\to\infty} v_1(z,\tau)e^{j\lambda\tau}, \quad b(\lambda,z) = \lim_{\tau\to\infty} v_2(z,\tau)e^{-j\lambda\tau}. \tag{12}$$

The NFT of $u(z,\tau)$, for fixed z, consists of the reflection coefficient $\rho(\lambda) = b(\lambda)/a(\lambda)$, for $\lambda \in \mathbb{R}$, and the discrete spectrum $\left(\lambda_j, \rho_j := \frac{b(\lambda_j)}{\frac{da}{d\lambda}(\lambda_j)}\right)$, where eigenvalues λ_j are the zeros of $a(\lambda,z)$ in the complex upper half-plane.

The evolution of these Jost scattering coefficients with respect to the standard NLSE (9) is given by

$$a(\lambda,z) = a(\lambda,0), \qquad b(\lambda,z) = b(\lambda,0)e^{j2\lambda^2 z} \tag{13}$$

The NFT of $q(z,\tau)$ with respect to (5) is now defined as the conventional NFT of the signal $u(z,\tau) = \sqrt{\frac{R(z)}{D(z)}}q(z,\tau)$. If $a(\lambda,z)$ and $b(\lambda,z)$ are the Jost scattering coefficients of $u(z,\tau)$, then their evolution with respect to (5) is given by

$$a(\lambda,z) = a(\lambda,0), \qquad b(\lambda,z) = b(\lambda,0)e^{j2\lambda^2 \int_0^z D(z)dz} \tag{14}$$

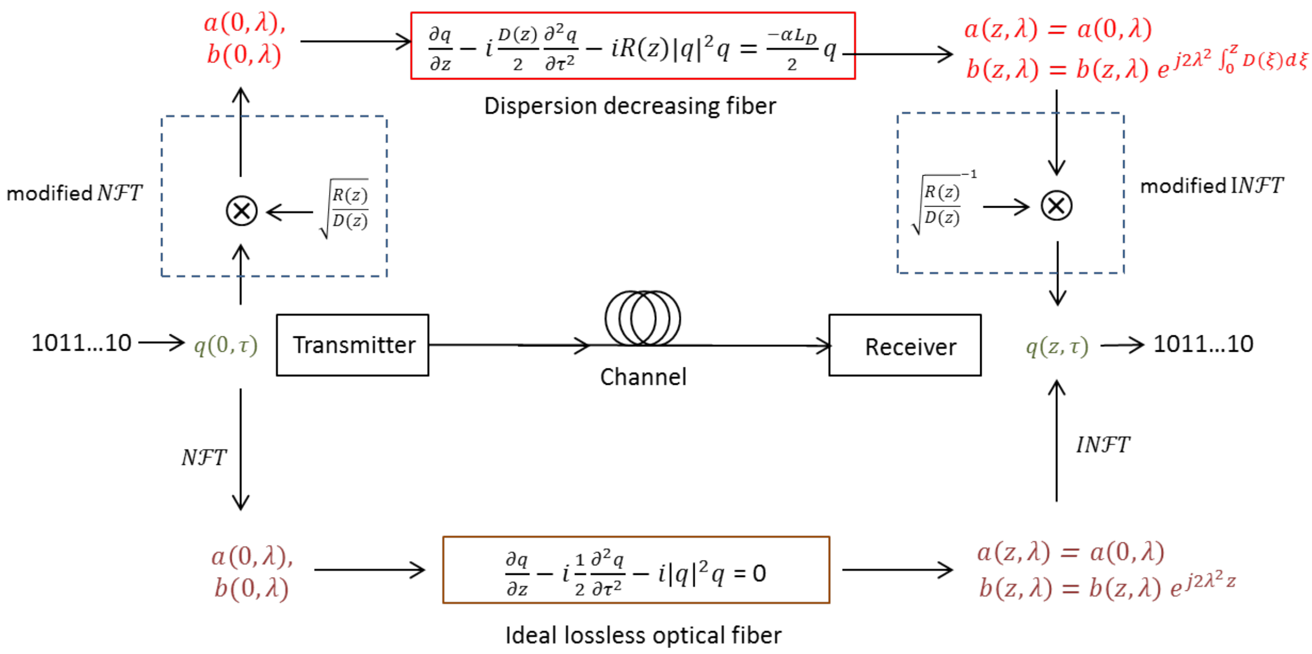

Figure 4: [Schematic of (I)NFT adapted for dispersion decreasing fiber.]

This equation allows us to recover the NFT of fiber input from that of the fiber output in a simple way. The schematic in figure 4 summarizes the modified NFT and explains the NFT based transmission system adapted for dispersion decreasing fiber. The modified NFT based transmission system with DDF makes the NFDM transmission exact.

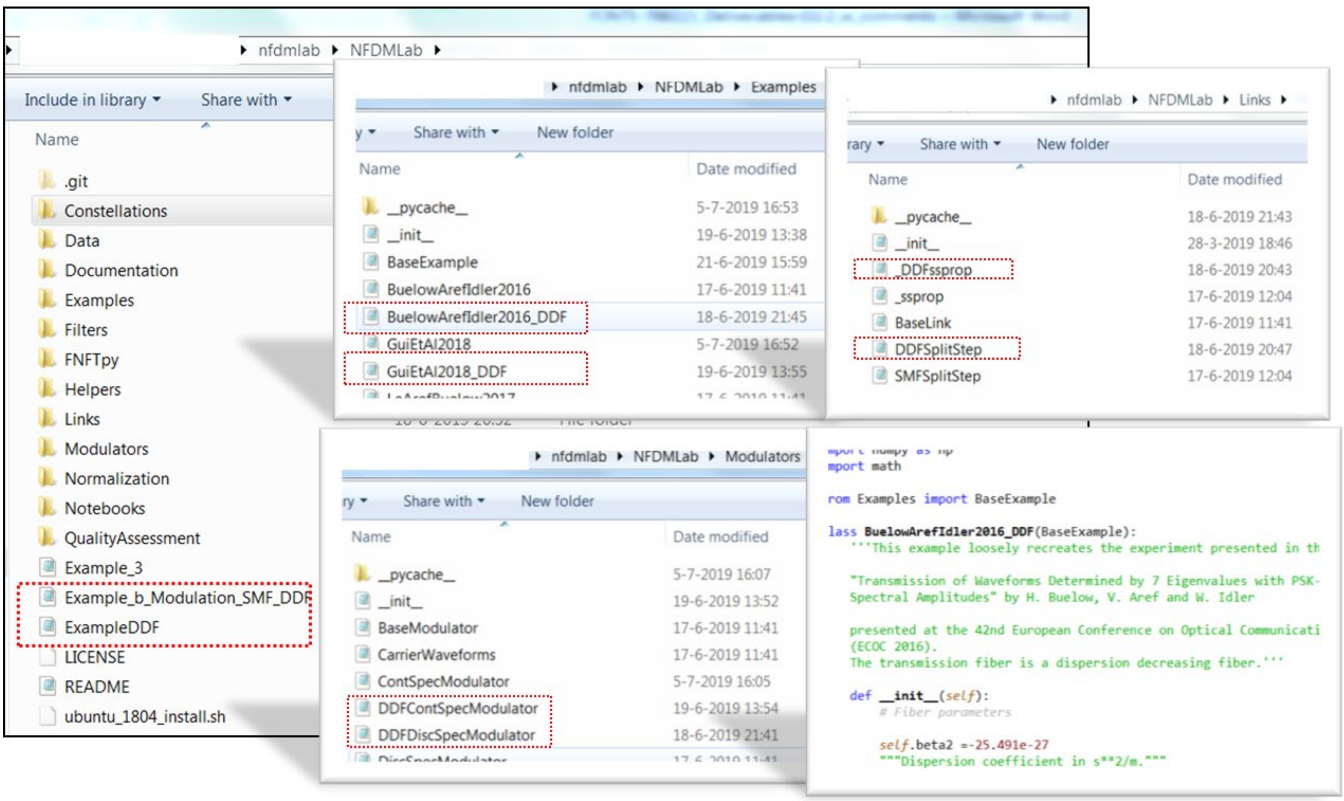

Figure 5: [Screenshots of NFDMLab showing files added to it.]

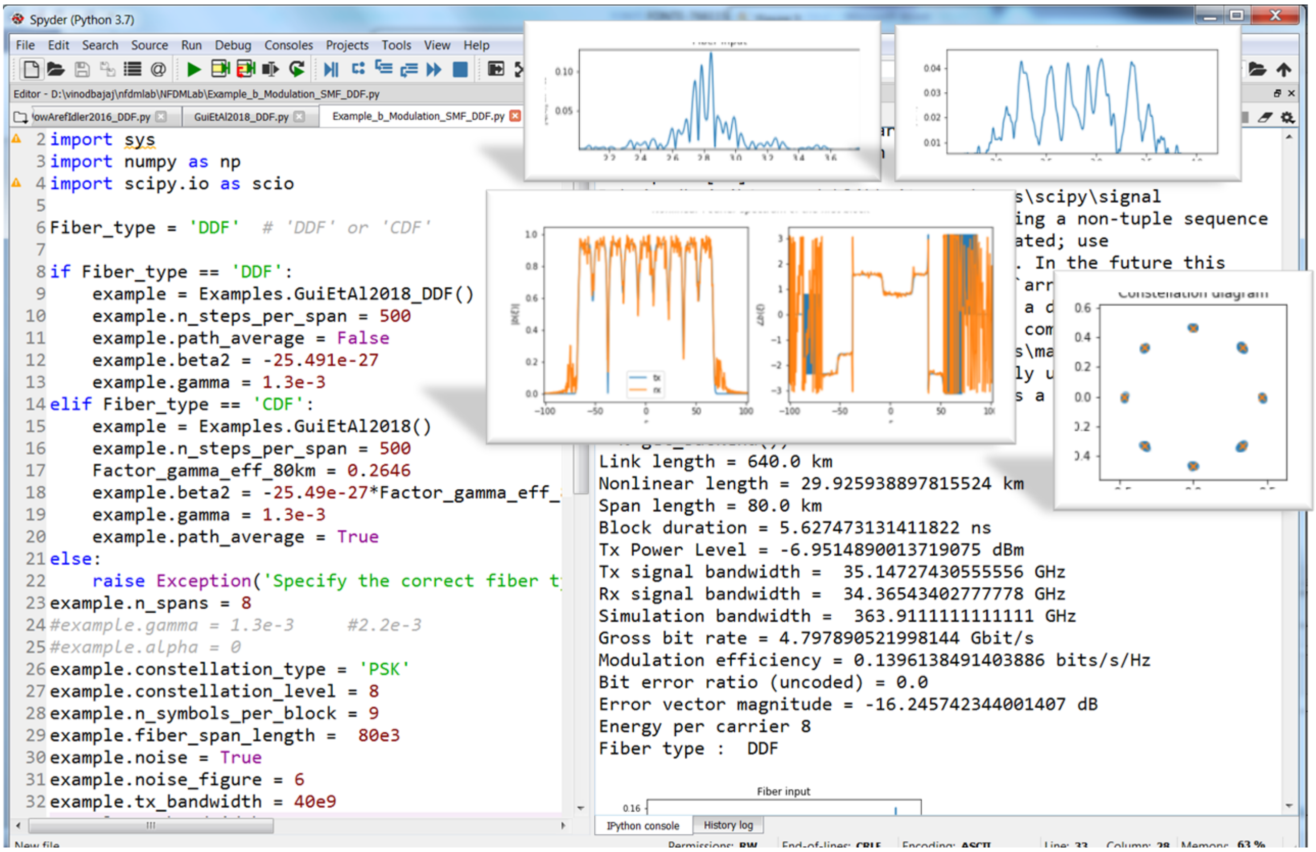

Figure 6: [Screenshot of execution of an example from NFDMLab.]

The following files were added to a development branch of NFDMLab GitHub repository, which are also shown in figure 5:

1. DDFsplitstep.py and _DDFssprop.py

_DDFssprop.py function simulates the propagation in a fiber with varying dispersion and nonlinear parameter using split step Fourier method. DDFsplitstep file, which implements EDFA amplification, is updated with a method to find profile of a DDF for a given fiber loss, initial dispersion and nonlinear parameter, such that the condition given in (6) is satisfied.

2. DDFContSepcMoulator.py and DDFDiscSepcMoulator.py

DDFContSepcMoulator.py and DDFDiscSepcMoulator.py modules implement back-propagation in nonlinear Fourier domain adapted to propagation in DDF, as per the relation is given by equation (14).

3. ExampleDDF.py along with BuelowArefIdler2016_DDF.py and Example_b_Modulation_SMF_DDF.py along with GuiEtAl2018_DDF.py are main execution example files for NFDM modulation with only discrete spectrum and only continuous spectrum respectively.
4. ContSepcMoulator.py and DDFContSepcMoulator.py files were modified to perform matched filter detection at receiver, thus improving detection in presence of noise.

The screenshot of execution of one of the implemented example is shown in figure 6. The above files are currently available in a development branch at https://github.com/FastNFT/NFDMLab/tree/DDF and it is planned to be integrated in main branch.

3 CONCLUSION

In this report we reviewed available software libraries for the computation of NFTs and simulation of NFDM systems. We discussed the use of dispersion decreasing fiber along with the modifications to the NFT that are required to make the NFDM transmission exact, thus avoiding the performance degradation due to the path average model. The functions to simulate propagation in DDF, modified (I)NFT and some other functionalities that have been contributed to the public open source NFDMLab software library were described.

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